Pricing of Longevity Risk: The Case of China*

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Abstract: In this paper we use the Lee-Carter model to quantify longevity risk and to investigate the effect of longevity risk on pension and insurance pricing and liabilities in the context of China. We calculate the expected present value of life annuities for retired Chinese males and females, taking into account stochastic mortality development, revealing a significant impact of longevity risk on annuity pricing.

Keywords: Longevity risk, China, Lee-Carter model

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1. Introduction

Like in most of the western world, the population of China has experienced a rapid aging over the past half century due to advances in public health, improved sanitation and personal hygiene, and general improvement in living standards (Lee, 2003; IMF, 2004). For example, the proportion of population aged 65 or older was only 4.41% in 1953 and 4.91% in 1982, respectively, but increased to 8.3% in 2008,[1] and by 2030 it will be more than doubled to 22% (James, 2002). Although age-specific death rates at all ages have declined exponentially at a constant rate in most developed countries (Tuljapurkat et al., 2000), it is the dramatically increased life expectancy at old age, along with lower fertility rates, that contributes to an increasing share of elderly people in the total population at rapid rate in both OECD countries and emerging economies, most notably in China (Visco and d'Italia, 2006). In 1981, for example, a 60-year-old Chinese female individual had a life expectancy of 17.90 years, whereas in 2000 a 60-year-old female had a life expectancy of 19.62 years, representing an increase of 1.09 months per year (Zheng, 2005), i.e., more than five minutes per hour. The trends in improving mortality among the elderly are significantly challenging public pension plans as well as private pension funds and life insurers. In the U.K. and the U.S alone these institutions' exposure to longevity amounts to 400 billion USD in 2007 (Loeys et al., 2007). One more year of life expectancy at age 65 is estimated to add at least 3% to the present value of the pension liabilities in the U.K. (Biffis and Black, 2009). In developing countries, including China, where pension systems are

[1] Data source: China Population and Employment Statistics Yearbook (2009).

underdeveloped,[1] these trends also significantly affect personal saving to fund retirement consumption. For example, consider a fairly-priced annuity with annual payoff ￥1 at the real interest rate of 3% in China. Then in 1981 the annuity price for 60-year female should have been ￥14.11, but could have increased by 7% to ￥15.11 in 2000. Without other retirement income, this means that the 60-year-old females in 2000 should have saved 7% more to finance their retirement consumption than in 1981.[2]

Though the views regarding the outlook for human longevity are still controversial (Antolin and Blommestein, 2007),[3] the general opinion from the experts tends to be the presence of upward trends in longevity. However, there is a large degree of uncertainty concerning the improvement magnitude, especially at older ages. From 1970 to 2000, the average increase in life expectancy of a 65-year-old male was 1.12 years/decade in the U.S. and 1.23 years/decade in the U.K., respectively, but the corresponding increase had only been 0.15 years over the previous decade in the U.S. and 0.17 years/decade over the previous century in the U.K. (Cocco and Gomes, 2008). China also experiences this uncertainty. According to Zhang (2005), in the 1980s the average increase in life expectancy of a 60-year-old Chinese male was 0.06 years per year, but increased to 0.09 years per year in the 1990s. Therefore, the major challenge faced by policy-makers, pension/insurance institutions, and individuals is

[1] In China, for example, only 201.37 million urban residents and 51.71 million rural residents are covered by public pension by 2007 (China Statistical Yearbook, 2008). According to the survey by the National Bureau of Statistics, whereas 68% city population aged 60 or over depends on pension as retirement income, the figure for rural population is only 4% (China Population and Employment Statistics Yearbook, 2009).
[2] Author's own calculation.
[3] For example, Olshansky et al. (2005) believe that there are natural limits to life expectancy, and suggest that the increase in life expectancy will slow down if not to stop; On the other hand, Oeppen and Vaupel (2002) argue that there are no limits to life expectancy and conclude from historical trends and age trajectories that longevity would keep increasing in the next decades.

not the trend in longevity itself, but rather the uncertain life expectancy in the future (De Waegenaere et al., 2010). When future life expectancy outcomes and mortality improvement turn out to be different from anticipated, longevity risk occurs.

Cocco and Gomes (2008) find that, when individuals use official period life tables, -which do not allow for future life expectancy improvement-, to make their retirement finance decision, the effect of longevity improvement on individual welfare can be significant. Moreover, the importance of longevity risk for the liabilities of private pension funds and annuity providers is that increasing portfolio size can only mitigate but cannot eliminate this risk. Therefore, several innovative solutions to longevity risk through the financial system, namely, reinsurance (Richards and Jones, 2004), natural hedging (Cox and Lin, 2007), and securitization (Cowley and Cummins, 2005), are being discussed. But all these solutions require better understanding of future mortality development.

In this paper we use the Lee-Carter model to quantify longevity risk and to investigate the effect of longevity risk on pension and insurance pricing and liabilities in the context of China. On the one hand, as the largest country in terms of population, China has been experiencing a faster decline in mortality among the elderly since the 1964-82 periods than the now low-mortality countries at comparable levels of overall mortality (Banister and Hill, 2004) and with its increasing prosperity these trends might be expected to continue. On the other hand, since most population in developed countries (at least) is covered by the public pension systems, the focus in these countries is more concentrated on the *distribution stage*; namely, depending on pension laws, people at retirement age receive their pension benefit either as lump sum, programmed withdrawal or as an annuity. On the contrary, much attention in China now is almost exclusively paid on the *accumulation stage*, namely how to increase the (public) pension coverage, with the ignorance of longevity risk by the public. These facts characterize the severity of longevity risk in China. Since most existing literature has scrutinized longevity risk in developed countries with fewer having sought to understand it in the context of the developing world, our first contribution is to fill in this gap. By quantifying this risk and assessing its impact on annuity pricing, we attempt to increase awareness and understanding of longevity risk by the public in the developing world, thus, contributing to the current public pension reforms and product design in China. Second, following De Waegenaere et al. (2010) and Hari et al. (2008), in this paper we distinguish diversifiable *individual mortality risk* and non-diversifiable *longevity risk*, investigating the impact of both risks on pension funds and annuity providers. Unlike its counterparts in the developed countries, the current public pension plans in China are decentralized to the local governments.[1] The relative small portfolio of each public pension plan means that these plans might face both risks. Third, since the Chinese statistical data on mortality are comparatively limited, we contribute to the existing literature on China's longevity risk by taking account of process risk as well as parameter risk. Finally, our research on the probability distribution of future mortality is important for China to respond to longevity risk through other innovative channels such as securitization.

[1] It is reported by *China Business News* (15 August 2008) that the public pension plans in China now are organized by around 2,000 entities. Even though people are free to relocate, their pensions are not allowed to transfer freely, especially between provinces.

The remainder of this paper proceeds as follows. In section two, we introduce the source of longevity risk and its impact on annuity pricing. In section three, we present the data and the Lee-Carter model, and then quantify the longevity risk taking account of process risk and parameter risk. We analyze the impact of longevity on annuity pricing in section four. Finally section five offers some concluding remarks.

2. Introduction to Longevity Risk

The uncertain mortality development may cause two kinds of risk, namely *longevity risk* and *individual mortality risk*.[1] According to Dahl (2004), *longevity risk* results from changes in the underlying mortality density, whereas *individual mortality risk* results from the random individual deaths with a fixed mortality density. For better understanding of the distinction between the two risks, see also De Waegenaere et al. (2010), we first introduce some scientific notation and terminologies of mortality.

2.1 Scientific Notations and Terminologies

The two basic building blocks of our projection of future life expectancy are one-year death probability, denoted by $q_{x,t}^{(g)}$, and the central death rate, denoted by $m_{x,t}^{(g)}$. The one-year death probability $q_{x,t}^{(g)}$ defines the probability that an x-year old person belonging to group g (female or male; rural or urban) will die within one year in year t.

The central death rate is defined by

$$m_{x,t}^{(g)} = \frac{D_{(x,t}^{(g)}}{E_{x,t}^{(g)}}, \tag{1}$$

[1] In some literatures, the two kinds of risk are also named as systematic longevity risk and unsystematic longevity risk, respectively. In order to highlight the non-diversification of the former, following De Waegenaere et al. (2010) we use *longevity risk* to indicate systematic longevity risk and *individual mortality risk* to unsystematic longevity risk.

where $D_{x,t}^{(g)}$ denotes the death number of people belonging to group g at age x in year t, while $E_{x,t}^{(g)}$, also called exposure, denotes the number of person years in group g at age x in year t.

Since both $D_{x,t}^{(g)}$ and $E_{x,t}^{(g)}$ can be observable from national statistics, we could obtain the one-year death probability $q_{x,t}^{(g)}$ from the central death rate $m_{x,t}^{(g)}$ (McCutcheon and Nesbitt, 1973). In the general case, this relationship is complicated, but can be simplified with appropriate additional assumptions. For example, under the assumption that the central death rate equals to the force of mortality,[1] we could establish the following relationship

$$q_{x,t}^{(g)} = 1 - \exp(-m_{x,t}^{(g)}). \tag{2}$$

With the one-year death probability, we could also obtain one-year survival probability, i.e., the probability that an x-year old individual belonging to group g survives at least another year in year t, by

$$p_{x,t}^{(g)} = 1 - q_{x,t}^{(g)}. \tag{3}$$

Under the assumption of *constant* time-independent mortality rates and one-year death probabilities over time, the one-year death (survival) probabilities would be independent of time and thus the subscript t can be suppressed. In this case, we could calculate the probability that a x-year old individual belonging to group g survives at least τ years (${}_{\tau}p_x^{(g)}$) and the corresponding remaining life expectancy for this individual ($e_x^{(g)}$) as follows.

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[1] The force of mortality, often referred to as the hazard function in other fields such as in reliability theory, is defined as $\mu_x = \lim_{\Delta x \to 0} \frac{P(x < X \le x + \Delta x \mid X > x)}{\Delta x}$ and specifies the instantaneous rate of death for x-year old people belonging to group g in year t, given that these individuals survive up to age x.

$$ {}_{\tau}p_x^{(g)} = \prod_{i=0}^{\tau-1} p_{x+i}^{(g)} , \tag{4} $$

$$ e_x^{(g)} = \sum_{\tau \geq 1} {}_{\tau}p_x^{(g)} , \tag{5} $$

where ${}_1p_x = p_x$. From the time point of year t, this individual is expected to die in $e_x^{(g)} + t$ years at the age of $e_x^{(g)} + x$.

However, the results above, based on the assumption of *constant* one-year death probabilities and mortality rates over time, might not be correct: neither are they constant over time, nor do they change in the same direction and at the same magnitudes for different cohorts. Figure 1 presents the mortality rates of selected age groups for different time periods, normalized to one for the year 1981.

[Insert Figure 1 here]

At least over longer time horizon, both Chinese females and males in these selected age groups experience significant mortality improvement, reflecting the increase in longevity over time. On the one hand, these improvements are different in terms of gender, ages, and years. On the other hand, at least to some extent, these improvements seem to be random, reflecting the stochastic chrematistics of the death probabilities. Since the death probabilities are not *constant* over time but rather *stochastic*, it's inappropriate to use (4) and (5) for calculating the remaining life expectancy of an x-year old individual belonging to group g in year t. With varying death probabilities, the survival probability of an x-year old individual belonging to group g for at least τ years in year t should follow

$$ {}_{\tau}p_x^{(g)} = \prod_{i=0}^{\tau-1} p_{x+i,t+i}^{(g)} . \tag{6} $$

The corresponding remaining life expectancy for this individual in year t should be calculated by

$$e_x^{(g)} = \sum_{\tau \geq 1} {}_{\tau} p_{x,t}^{(g)} . \tag{7}$$

Both (6) and (7) need future death probabilities that are unobservable for the current period. Thus, when using current death probabilities rather than the projected ones, the expected life expectancy as well as the discounted value of pension liabilities might be underestimated. See, for example, Hari et al. (2006).

2.2 Significance of Longevity risk

Assuming a finite number of scenarios for the evolution of future mortality probabilities, many studies (Olivieri, 2001; Coppola et al., 2000, 2003a, and 2003b) find that even when the size of portfolio is increased, the *longevity risk* cannot be diversified and does not disappear, whereas the *individual mortality risk* is diversifiable. In most developing countries the pooling size of pensions is relatively small. Thus, unlike their counterpart in developed world, the pension systems in developing countries typically might face both risks.

In order to demonstrate the non-diversifiable characteristics and significance of *longevity risk* and its distinction from *individual mortality risk*, we consider a pension plan composed of N x-year old immediate lifetime annuitants belonging to group g in year t. For simplicity, we assume that each annuitant gets one Chinese Yuan per year after retirement conditional on his/her survival, with a constant risk-free interest rate r. Thus, in year $t+\tau$ ($\tau \geq 1$) the present value of the future payment to annuitant i should follow

$$Y_i = \sum_{\tau \geq 1} 1_{i,t+\tau} \frac{1}{(1+r)^\tau}, \tag{8}$$

where $1_{i,t+\tau}$ donates the dummy variable with value equal to one if annuitant i is still alive in year $t+\tau$.

We first only consider *individual mortality risk*, namely that the future mortality improvements are known with certainty. In year t, the expected present value of the future payment to annuitant i is thus given by

$$A_{x,t} = \sum_{\tau \geq 1} E(1_{i,t+\tau}) \frac{1}{(1+r)^\tau} = \sum_{\tau \geq 1} {}_\tau p_{x,t} \frac{1}{(1+r)^\tau}. \tag{9}$$

According to the pooling argument, $A_{x,t}$ should be the fair price of this annuity and the fair price of Y_i should be the same as the fair price of $\frac{1}{N}\sum_{i=1}^{N} Y_i$. Under the assumption of independent annuitants, we can get the following variance

$$Var(\frac{1}{N}\sum_{i=1}^{N} Y_i) = \frac{\sigma^2}{N}, \tag{10}$$

where we take $\sigma^2 = Var(Y_i)$ and $\mu = E(Y_i)$.

Obviously, with increasing pooling size, the variance of $\frac{1}{N}\sum_{i=1}^{N} Y_i$ approaches zero and thus the risk free and its fair price equals to its expected present value, and there is no risk premium accordingly. With certain future death probabilities, pension plans and insurance companies only face *individual mortality risk* that can be eliminated by pooling.

When the future death probabilities are uncertain, however, *longevity risk* becomes dominant. We continue with the pension plan composed of N x-year old immediate lifetime annuitants belonging to group g in year t, given the set of future death rates in year t by $f_t = \{q_{x,t+\tau}^{(g)} \mid \tau \geq 1\}$. We follow the assumption of independent annuitants in *individual mortality risk* above but have different mean and variance both depending on f_t, i.e. $\mu(f_t)$ and $\sigma^2(f_t)$. Thus, (10) should be replaced by

$$Var(\frac{1}{N}\sum_{i=1}^{N} Y_i) = E[Var(\frac{1}{N}\sum_{i=1}^{N} Y_i) \mid f_t] + Var[E(\frac{1}{N}\sum_{i=1}^{N} Y_i \mid f_t] = \frac{E[\sigma^2(f_t)]}{N} + Var[\mu(f_t)]. \tag{11}$$

With increasing pooling size, the first term on the right side of (11) can still be eliminated, but the second term continues to exist, independently of N. With the existence of *longevity risk*, the pooling argument cannot eliminate mortality risk any more and a risk premium should be included into the pricing of financial products whose payoffs depend on the future mortality development.

In the context of China, on the one hand, the underdeveloped pension systems and low coverage might mean that both *individual mortality risk* and *longevity risk* exist. On the other hand, the incomplete market and non-diversifiable characteristics make the pricing of longevity risk and risk management more difficult in China than in developed countries.

3. Lee-Carter Models and Longevity Risk in China

In this paper, we only discuss purely statistical mortality models, without considering other exogenous demographic or epidemiological factors, because pension funds and annuity providers are much more interested in "all-cause" mortality (Hari, 2006).

Generally, the stochastic mortality models are more parsimonious,[1] trying to

[1] There are two types of mortality models: deterministic model and stochastic model. Starting from De Moivre

explain the death rates with unobserved latent factors. Actually, when looking at sequences of mortality curves over a relatively long horizon, we can easily find that they change unpredictably, not only from one period to another, but also over the long term, though they do exhibit a general trend. Thus, it is more accurately to model the mortality in a stochastic fashion. Among these stochastic models, the Lee-Carter model (1992) has become the “leading statistical model of mortality in the demographic literature” (Deaton and Paxson, 2004) and, along with its extensions, has been widely applied for many countries for its simplicity and robustness in the context of linear trends in age-specific death rates, for example, Japan (Wilmoth, 1993), G7 countries (Tuljapurkar et al, 2000), Australia (Booth et al, 2000, 2002), England and Wales (Renshaw and Haberman, 2003), Belgium (Brouhns et al, 2002), and the Netherlands (Hari et al., 2008; Stevens et al., 2010). However, the existing literature using the Lee-Carter model for developing countries, including China is rather limited and incomplete, partly due to the unavailability of data.

3.1 Lee-Carter Model

According to Lee and Carter (1992), the log central death rate of the x-year-old persons in year t, $\ln(m_{x,t})$, is determined by a common latent factor κ_t, with an age-specific level parameter, α_x, and an age-specific sensitivity parameter, β_x. Mathematically, the model can be expressed as follows:

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}, \quad (12)$$

(1724)1, the deterministic approach (Gompertz, 1825; Makeham, 1860; Heligman and Pollard, 1980) typically only considers the age dimension, though recent models try to fit mortality rates in both of age and of time dimension. However, since this kind of approach usually does not take account of uncertainty and also the accurate in-sample fit is translated into only small prediction intervals, it has not seemed to be very realistic in practice.

where the white noise error terms, $\varepsilon_{x,t}$, represent the transitory non-systematic shocks.

Obviously, the OLS method cannot be applied to the Lee-Carter model because none of the variables on the right hand of equation (12) are observable. In order for a unique solution, Lee and Carter first normalize the sum of β_x terms to unity and κ_t terms to zero, i.e., $\sum_x \beta_x = 1$ and $\sum_\kappa \kappa_t = 0$, and get the value of α_x since it becomes the average value of $\ln(m_{x,t})$ over time. Then they use a two-stage approach to solve this under-identification problem. The singular value decomposition (SVD) approach is used in the first stage for the matrix of $\ln(m_{x,t}) - \hat{\alpha}_x$ to get estimates of κ_t and β_x. In the second stage, given the value of $\hat{\alpha}_x$ and $\hat{\beta}_x$, $\hat{\kappa}_t$ is re-estimated by iteration until the implied death number equals the actual death number such that

$$\sum_x D_{x,t} = \sum_x \left[E_{x,t} \exp(\hat{\alpha}_x + \hat{\beta}_x \hat{\kappa}_t) \right]. \qquad (13)$$

Nevertheless, in the first stage of this two-step procedure above a weighted singular value decomposition could also be used (Wilmoth, 1993). Moreover, Lee and Miller (2001) proposed using a matching on the basis of observed and modeled life expectancy rather than the matching according to (13). In addition, in order to avoid the violation of the assumption of constant α_x and β_x, Booth, Maindonald, and Smith (2002) suggest using statistical techniques to select an appropriate sample period.

Originally, Lee and Carter find that κ_t satisfies a random walk with drift process as:

$$\kappa_t = \kappa_{t-1} + c + \xi_t \qquad (14)$$

where the white noise term, ξ_t representing permanent shocks, is assumed to be independent of $\varepsilon_{x,t}$ and to follow a normal distribution with mean zero and variance of σ_{ζ}^{2}. With standard statistical or econometric time-series techniques, the parameters in (14) can be estimated. However, the ARIMA process of κ_t for other countries might be different from (13). Thus, standard statistical procedures should be applied to find an appropriate ARIMA model for the time series of κ_t (Liu, 2008).

In this way, the systematic path of the central mortality rate of the x-year-old persons in year t satisfies:

$$\widehat{m}_{x,t} = \exp(\widehat{\alpha}_x + \widehat{\beta}_x \widehat{\kappa}_t). \tag{15}$$

In order for the projection of future mortality, we firstly need to forecast the future values of $\widetilde{\kappa}_{T+\tau}$ (T is the final year of the sample) and then the systematic path of future central mortality rate by

$$\widehat{m}_{x,t+T} = \exp(\widehat{\alpha}_x + \widehat{\beta}_x \widetilde{\kappa}_{T+\tau}), \tag{16}$$

In order to avoid a jump-off bias, Lee and Miller (2001) alternatively propose using the observed (raw) central death rate of the final year in the sample as a jump-off value to predict the future central death rates such that

$$\widetilde{m}_{x,t+T} = m_{x,t} \exp(\widehat{\beta}_x (\widetilde{\kappa}_{T+\tau} - \widehat{\kappa}_T)). \tag{17}$$

With the assumption that the force of mortality does not change during a year, i.e., $m_{x+s,t+s} = m_{x,t} (0 \le s < 1)$, the survival probability of one more year for one x-year-old person at time t is calculated by (2) and (3). From (6) and (7), we can obtain the projection of life expectancy at different ages. Undoubtedly, there might be several risks in our projection resulting from the scholastic nature of $\widetilde{\kappa}_{t+T}$. First, since neither the true value of $\widetilde{\kappa}_{t+T}$ nor its distribution is known at time T, the *process* *risk* might arise. Second, limited sample size and measurement error might cause inaccurately estimated coefficients of α_x, β_x and κ_t, which generates *parameter risk*. In addition, without knowing exactly the true distribution of $\hat{\kappa}_{t+T}$, but having to model it, there might cause *model risk*. For methods quantifying these risks, see Koissi et al. (2006), Renshaw and Haberman (2008).

3.2 Data and Estimation Results

Our data include 15 yearly observations of age-specific death rates for both males and females in China from 1994 to 2008, provided by the China Population Statistical Yearbooks and the China Statistical Yearbooks compiled by the National Bureau of Statistics of China. The age-specific death rates are tabulated from age 0, 1, 2, up to age group 85 and over. Figure 2 shows the logarithm of the raw central death rates of Chinese males and females for ages zero to 84 and age class 85 and over for the sample period 1994-2008. Like in most countries, the mortality pattern for each year in China firstly starts rather high for newborn infants and goes down at around age 15, then increasing again with the accident hump at around age 20-25.

[Inset Figure 2 Here]

Using the singular value decomposition (SVD) approach, we firstly estimate the values of α_x, β_x and κ_t, respectively.

[Inset Figure 3 Here]

Figure 3 plots the estimated α_x (left panel) and β_x (right panel), respectively. Since α_x is the average value of $\ln(m_{x,t})$ over time, it can be interpreted as the mean age profile of mortality. The estimated α_x show the similar mortality pattern as figure 2. Moreover, for most age groups the estimated α_x for females is smaller than or equal to that for males, explaining the fact that females, on average, face a longer life expectancy than males. As the loading factor, β_x measures the age-specific response to the changes in the latent factor, κ_t. For example, a low (high) value of β_x represents slowly (rapidly) decrease of mortality at specific age if κ_t declines over time. Our estimation results (see right panel), after being smoothed, show that, in China, females aged between 10 and 50 are more sensitive to mortality movement but less so in the other age groups. Moreover, the smoothed curves follow different patterns for males and females, with the hump at around 10-30 for females. It seems that older males experience much bigger mortality improvement κ_t declines over time during the sample period.

[Insert Figure 4 and Table 1Here]

Figure 4 plots the estimated κ_t, which, according to our expectation, shows the decreasing trends over time generally. In order to determine the appropriate ARIMA model for the time series of κ_t, we use the Augmented Dickey-Fuller test and the Phillips-Perron Test to check stationarity. Panel A of table 1 shows the test results and confirms the existence of a unit root for the κ_t processes in level, but stationarity after first differencing. Thus, the κ_t processes for both Chinese males and females seem to be integrated of order one. Moreover, from panel B of table 1 we conclude that both the male and female κ_t processes follow a random walk, which differs from previous findings. For example, Yin (2005) finds that the male process is an ARIMA (0, 1, 1) process. We use the OLS method to estimate (14) and the estimation results of κ_t process for males and females are shown by (18) and (19), respectively.

$$\kappa_t^{(m)} = -1.5592 + \kappa_{t-1}^{(m)}, \text{ with } \hat{\sigma}_\xi^{(m)} = 6.2647 \tag{18}$$

$$\kappa_t^{(f)} = -4.0662 + \kappa_{t-1}^{(f)}, \text{ with } \hat{\sigma}_\xi^{(f)} = 6.5438 \tag{19}$$

[Insert Table 1 Here]

Based on the results above, we can use project future values of $\tilde{\kappa}_{t+T}$ with (18) and (19) for males and females, respectively, and then calculate projected one-year death probabilities and life expectancies at different ages according to the relevant equations above.

We now show the longevity risk resulting from *process risk* and *parameter risk* through predicting the logarithm of the central death rate beginning from 2009, the first year after our sample period. Due to the random walk of the estimated κ_t, we can use the Girosi and King (2006)-variant of the Lee-Carter model to illustrate these risks (see appendix) because the T-asymptotic characteristics of the estimator based on this variant imply that making predictions as well as quantifying the longevity risk becomes a standard exercise in statistics or econometrics (De Waegenaere et al., 2010). Figure

We show the observed and 15-year ahead prediction of the logarithm of the central death rate for 60-year old Chinese with parameter risk and process risk in figure 5 and figure 6, respectively. The prediction begins from 2009, the first year after our sample period. In figure 5, the two cases, i.e., only parameter risk and the combination of parameter risk and process risk, are taken into account, whereas figure 6 considers only process risk and the combination of process and parameter risk, using computing 95% confidence intervals. Both figures show clearly the downward trends not only in-sample but also out-of-sample, predicting mortality improvements in the future. For example, at the beginning of our sample (1994), the one-year death probability calculated based on (2) for one 60-year old male is 0.0141, but decreases to 0.0115 in 2025, representing around 18% in just 30 years. However, figure 5 and 6 also show the high uncertainty about future mortality movement in terms of direction and magnitude.

[Insert Figure 5 and 6 here]

3.3 Pricing of Longevity Risk

In this section we investigate the impact of longevity risk on public pension plans as well as private pension funds and life insurers by calculating the expected present value of a life annuity in different scenarios through simulation. We assume that each annuitant gets one Chinese yuan per year after retirement, conditional on his/her survival, with a constant risk-free interest rate r or under term structure of interest rate of government bond. Thus, in year $t+\tau$ $(\tau \geq 1)$ the present value of the future payment should follow (8). In order to highlight the impact of scholastic death probabilities on the annuity price, we also calculate the expected present value of a life annuity under the assumption of constant one-year death probabilities based on (4) and (5).

[Insert Table 2 Here]

Table 2 presents the simulation results for the annuity price in different scenarios with corresponding 95% confidence intervals in parenthesis. Column (1) and (5) show the expected present value of a life annuity for 60-year old Chinese males and females in 2009 under constant death probabilities, respectively. Without accounting for the stochastic death probabilities and future mortality improvement, these results are unsurprisingly lower than those in the framework of stochastic death probabilities. For example, with interest rate of 3%, the life annuity price for 60-year old female in 2009 is only 15.17 based on the assumption of constant death probabilities; however, when stochastic mortality development is taken into account, the same person should pay, at least, 15.94 yuan or around 5% more for the same annuity product. Therefore, without taking into account longevity risk and its randomness when designing pension systems or products, the impact of longevity risk on risk management would be substantial.

4. Conclusions

In this paper we use the Lee-Carter model to quantify longevity risk and to investigate the effect of longevity risk on pension and insurance pricing and liabilities in the context of China. Unlike previous research, we find that both the latent factor process for Chinese males and females also follows a random walk. In addition to *process risk*, resulting from the unknown distribution of the latent factor in the future, we also take into account the impact of *parameter risk* on our projection of future mortality development. Though the future mortality development shows a strong downward trend, it also presents substantial uncertainties when *process risk* and *parameter risk* are involved. In order to investigate the impact of longevity risk on pension plans and insurance companies, we simulate the expected present value of life annuity for

60-year old Chinese males and female beginning from 2009. As comparison, we also calculate the corresponding annuity price under constant death probabilities for comparison. Our simulated results show that, without taking into account the stochastic mortality development in the future, the pricing of life annuity products would be underestimated, significantly challenging public pension plans as well as private pension funds and life insurers.

As the world's largest country in terms of population, China has experienced a rapid aging over the past half-century and thus the Chinese government is reforming its pubic pension system to meet the urgent challenges of an ageing society. Since (public) pension coverage is still low in China, compared with other developed economies, much attention in China now is almost exclusively paid to the *accumulation stage*, with the ignorance of longevity risk by the public and policy makers. However, this paper reveals the significant impact of longevity risk on risk management and pension/annuity pricing. Thus, increasing awareness and understanding of longevity risk by the public, especially the policy makers, would contribute to the current public pension reforms and product design in China.

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Appendix: The Girosi and King (2006)-variant of the Lee-Carter model

First of all, let

$$l_t = \begin{pmatrix} \ln(m_{1,t}) \\ \vdots \\ \ln(m_{ma,t}) \end{pmatrix},$$

where ma stands for the maximum age.

Then, let

$$\alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_{ma} \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_{ma} \end{pmatrix}, \text{ and } \quad \varepsilon_t = (\varepsilon_{1,t}, \cdots, \varepsilon_{ma,t})$$

Now, from

$$l_t = \alpha + \beta\kappa_t + \varepsilon_t \quad \text{and} \quad \varepsilon_t = \mu + \varepsilon_{t-1} + \delta_t$$

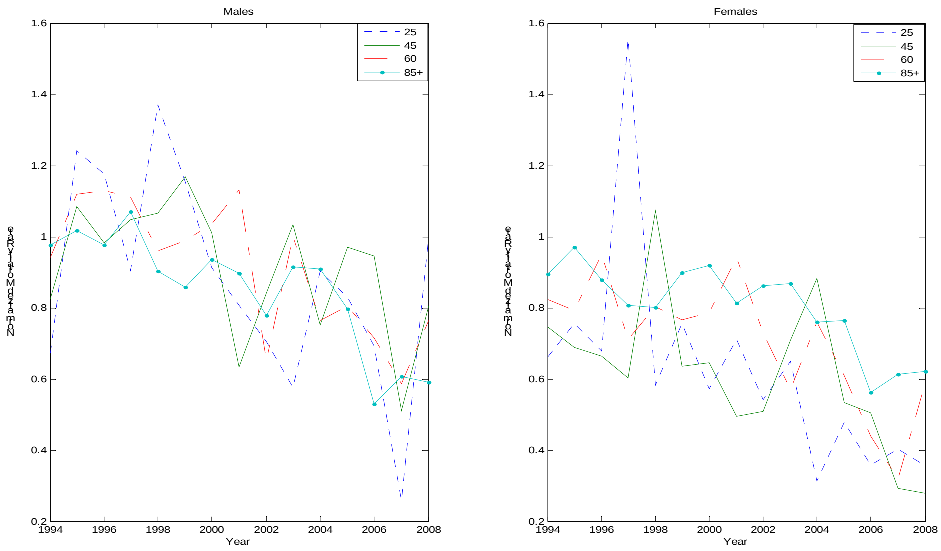
the Lee-Carter model can be rewritten as

$$l_t = \theta + l_{t-1} + \zeta_t$$

where $\theta = \beta\mu$ and $\zeta_t = \beta\delta_t + \varepsilon_t - \varepsilon_{t-1}$

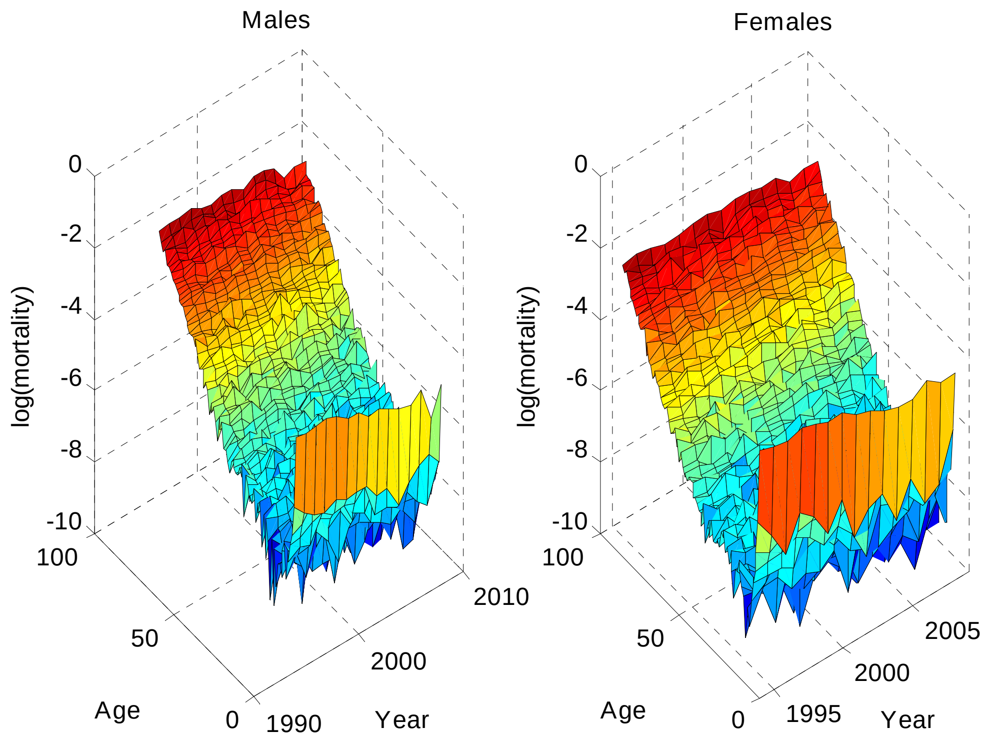
Now, we can easily estimate the model, make prediction and quantify the longevity risk.

Figure 1: Normalized Death Rate for Selected Age Groups



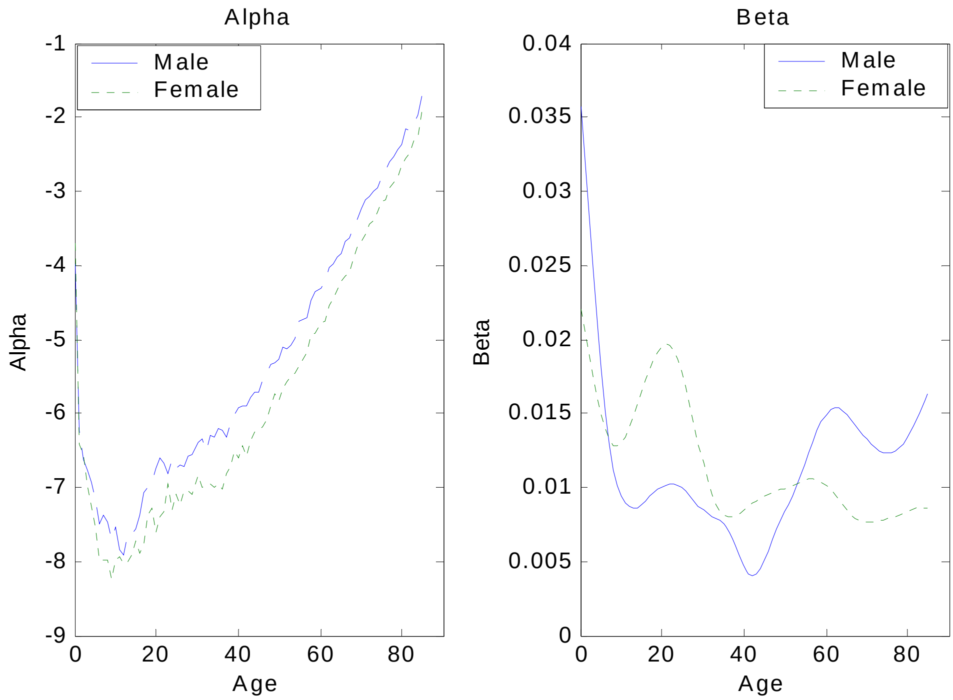
This figure plots the observed death rates for Chinese males (left) and Chinese females (right), for selected age groups and for different time periods, normalized to one for year 1981. The data originates from the China Population Statistical Yearbooks and the China Statistical Yearbook compiled by the National Bureau of Statistics of China.

Figure 2: Logarithm of Raw Central Death Rates in China, 1994-2008



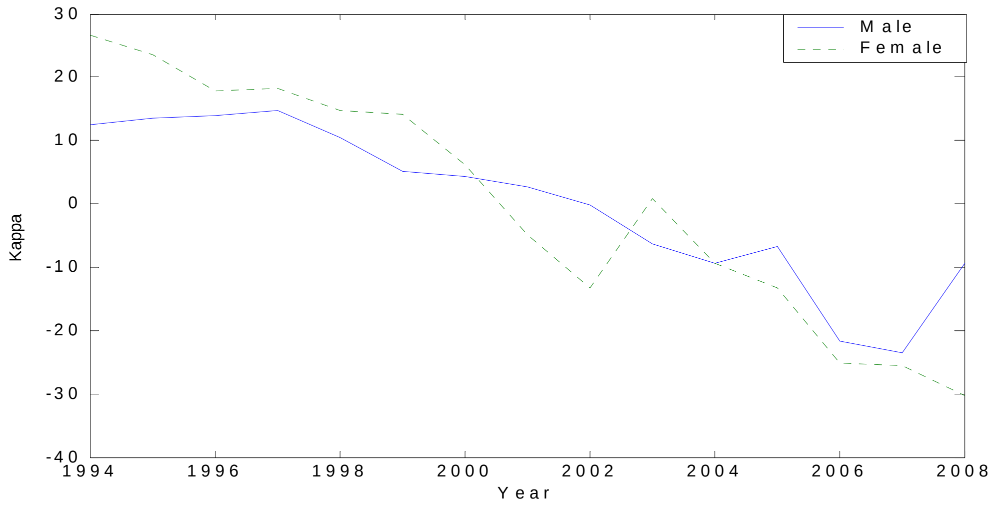
This figure plots the logarithm of raw central death rates during the 1994-2008 period for Chinese males (left) and Chinese females (right) from age 0, 1, 2, up to age group 85 and over. The data originates from the China Population Statistical Yearbooks and the China Statistical Yearbooks, both of which are compiled by the National Bureau of Statistics of China.

Figure 3: Estimated α_x and β_x



This figure presents the estimated α_x (left panel) and β_x (right panel, smoothed using cubic B-splines) for both Chinese males and females, from age 0, 1, 2, up to age group 85 and over.

Figure 4: Estimated κ_t



This figure plots the estimated κ_t for Chinese males and females for 1994-2008 period.

Table 1: Unit Root Test and Model Selection for κ_t

Panel A: Unit Root Test in Level and first Difference

	Male		Female	
Statistics	t-Stat	Prob.	t-Stat	Prob.
Augmented Dickey-Fuller Test in Level	1.1701	0.9952	-0.3910	0.8859
Phillips-Perron Test in Level	-0.8662	0.7673	0.1935	0.9613
Augmented Dickey-Fuller Test first Difference	-4.7196	0.0038	-3.8698	0.0167
Phillips-Perron Test in first Difference	-1.4418	0.5296	-8.8622	0.0000

Panel B: Autocorrelation and Partial Correlation of First Difference

	Male				Female			
	AC	PAC	Q-Stat	Prob.	AC	PAC	Q-Stat	Prob.
1	-0.071	-0.071	0.0871	0.768	-0.324	-0.324	1.8042	0.179
2	-0.015	-0.021	0.0915	0.955	-0.071	-0.197	1.8997	0.387
3	-0.032	-0.035	0.1126	0.990	-0.124	-0.250	2.2122	0.530
4	0.022	0.017	0.1235	0.998	0.110	-0.057	2.4838	0.648
5	0.023	0.025	0.1365	1.000	0.001	-0.032	2.4839	0.779

Table 2: Simulation Results for Life Annuity Price, 60-year Male and Female in 2009

Panel A: Flat Rates								
r	Male	Female						
	Constant Death Probabilities (1)	Process Risk (2)	Parameter Risk (3)	Process & Parameter Risk (4)	Constant Death Probabilities (5)	Process Risk (6)	Parameter Risk (7)	Process & Parameter Risk (8)
0.01	16.1499	17.0634	17.0692	17.0393	18.4207	19.5873	19.5933	19.5487
	(15.4562-16.8230)	(15.2385-18.6268)	(15.2834-15.6593)	(15.2418-18.6116)	(18.0970-18.7370)	(18.8202-20.2484)	(18.8335-20.2584)	(18.7609-20.2403)
0.02	14.7287	15.4900	15.4875	15.4627	16.6668	17.6302	17.6321	17.6043
	(14.1348-15.3001)	(13.9594-16.8147)	(13.9982-16.7869)	(13.9752-16.7864)	(16.3872-16.9318)	(16.9798-18.1824)	(17.0120-18.1826)	(16.9651-18.1654)
0.03	13.5066	14.1282	14.1372	14.1310	15.1659	15.9578	15.9660	15.9440
	(12.9968-14.0006)	(12.8373-15.2423)	(12.8533-15.2620)	(12.8629-15.2388)	(14.9342-15.3938)	(15.4330-16.4217)	(15.4361-16.4260)	(15.4036-16.4242)
0.04	12.4384	12.9832	12.9722	12.9592	13.8722	14.5351	14.5395	14.5161
	(11.9969-12.8631)	(11.9832-13.9337)	(11.8954-13.9082)	(11.8611-13.9187)	(13.6733-14.0678)	(14.0894-14.9274)	(14.1008-14.9306)	(14.0553-14.9237)
0.05	11.5180	11.9737	11.9694	11.9636	12.7542	13.3152	13.3085	13.2910
	(11.1309-11.8856)	(11.0747-12.7768)	(11.0666-12.7757)	(11.0427-12.7690)	(12.5797-12.9192)	(12.9426-13.6392)	(12.9243-13.6401)	(12.9010-13.6332)
Panel B: Term Structure								
	Constant Death Probabilities (1)	Process Risk (2)	Parameter Risk (3)	Process & Parameter Risk (4)	Constant Death Probabilities (5)	Process Risk (6)	Parameter Risk (7)	Process & Parameter Risk (8)
Term	13.0671	13.6157	13.6158	13.5950	14.5559	15.2392	15.2362	15.2171
Structure	(12.6048-13.5097)	12.5002-14.6237)	(12.4963-14.5842)	(12.4556-14.5783)	(14.3468-14.7542)	(14.7850-15.6441)	(14.7758-15.6408)	(14.7459-15.6342)

This figure table presents the simulated annuity price for 60-year old Chinese males and females in 2009 under different scenarios. Panel A is based on the flat rates and Panel B on term structure of China's government bond 24th May, 2010.