Cover Page

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I would like to submit the paper to the special issue with the *Journal of Chinese Economics and Business Studies*.

I am a first year PhD student at Department of Economics, University of Oxford and would like to participate in the Best Paper competition. My supervisor is Professor David Vines, Economics Department and Balliol College, Oxford University; Centre for Applied Macroeconomic Analysis, Australian National University; and CEPR

Understanding Chinese Renminbi Appreciation: A Microfounded Model with Imperfect Capital Mobility

Sheung Kan Luk*

June 13, 2010

Abstract

This paper uses a two-country DSGE model to study how the growing Chinese current account surplus, and the associated undervaluation of the renminbi, might be resolved. To reflect the importance of capital controls in China, the model assumes both imperfect substitutability between home and foreign financial assets and home bias in asset portfolios. Once the correction process begins, the initial jump in the real exchange rate is less than the long run change, and a transient period of current account imbalance reallocates wealth across countries. This is what is required by the less-than-perfect international mobility of capital. The size of the initial exchange rate jump, and the subsequent speed of appreciation, are governed by the degree of capital mobility. We calibrate the model and use it to capture that kind of positive productivity shocks and demand shocks which the Chinese economy is currently experiencing. The results suggest that it might be possible to gradually remove the current disequilibrium, at a rate of a real appreciation of the Chinese currency of about 5-8% per year.

*Economics Department and Lady Margaret Hall, Oxford University (Email: sheung.luk@economics.ox.ac.uk). This paper is extracted from my thesis submitted in partial fulfillment of the requirements for the degree of Master of Philosophy in Economics in University of Oxford. I am grateful to Andres Velasco and David Vines for helpful discussions. Financial support from the Sir Edward Youde Memorial Fund Council is gratefully acknowledged. All errors remain my own.

1 Introduction

This paper aims to make a contribution to understanding how the growing Chinese current account surplus, and the associated undervaluation of the renminbi, might be resolved.[1]

Since 1994, the Chinese yuan has been convertible but fixed with the U.S dollars. There was a period of overvaluation during the 1997-98 Asian financial crisis, but after the accession of China into the WTO in 2001, owing to a combination of a steady shift of world demand towards Chinese productions and a rising Chinese productivity, the Chinese yuan became undervalued and foreign exchange reserves began to build up. By 2009, foreign exchange reserves have ballooned to about 2 trillion U.S dollar.[2]

Economists and the wider public alike are concerned about the undervaluation of the Chinese yuan and global imbalance. There are various suggestions on how to revalue the yuan exchange rate, including the two-step adjustment proposal by Goldstein and Lardy (2003, 2008) and some others. However, a large proportion of research on Chinese exchange rate regime, as we are aware of, are either empirical or descriptive.[3] This paper approaches the issue from another direction. It uses a simple two-country dynamic stochastic general equilibrium (DSGE) model to analyse U.S-China exchange rate adjustment when the economies are hit by demand and productivity shocks.

It is natural to consider a portfolio balance type model which assumes that assets are not perfectly indistinguishable. As a result, the uncovered interest parity (UIP) condition does not hold in the short run and return on assets in different countries will not be equal. To replace the UIP, we follow Blanchard Giavazzi and Sa (2005) and Kirsanova, Kuralbayeva and Vines (2008) (henceforth KKV) to model imperfect asset substitutability. People are willing to hold assets with lower return because the asset is in excess demand due to home asset bias. The exchange rate then has to clear both the goods market and the asset market, so there will be an initial jump. With imperfect capital mobility, exchange rate does not jump to the long-run equilibrium immediately, which gives rise to current account imbalances and the associated stock-flow dynamics. In the short run, current account imbalances relocate wealth between countries which leads to gradual exchange rate adjustment to the steady state. The sluggish adjustment in exchange rate is in contrast with models which use UIP (See Clarida, Gali and Gerler 2001 for example.).

We would like to make it claer up front that this work is not an attempt to be a

[1] The renminbi is the name of the currency in People's Republic of China. The principle unit is the yuan. In the paper, these two names may be used interchangeably.

[2] Source: Xinhua News Agency. 'China's Foreign Reserves Hit $1.95 Trillion At End Of March'. 11th April, 2009.

[3] There are exceptions. Blanchard and Giavazzi (2005) construct a model with tradable, non-tradable and agricultural sector. However, the model is not based on utility maximisation. Devereux and Genberg (2007) use a two-country DSGE framework to analyse the relations between renminbi appreciation and current account adjustment. We focus on the exchange rate appreciation considering endogenous portfolio allocation in the presence of imperfect capital mobility.

full analysis of the global imbalance. A deeper issue which we abstract from is the high savings rate in China relative to the U.S. The high savings rate leads to a saving and investment gap which gives rise to the current account imbalances and real exchange rate adjustments that this paper discusses. A more complete analysis would therefore involve understanding the disparity in the saving behaviours in an endogenous framework and investigating shocks that are more realistic and relevant, which this paper does not deal with.

We describe the main findings of this paper. Imperfect capital mobility leads to dynamic exchange rate and current account adjustments. The degree of capital mobility determines the size of initial jump and speed of subsequent exchange rate adjustment. The more mobile capital is, the larger is the initial appreciation in the yuan and the more gradual the later adjustment. Current account imbalance will persist until the adjustment is complete. Exchange rate movement also depends on the size of the shocks. We find that demand shocks and productivity shocks move the real exchange rate in different directions and that exchange rate movements are more sensitive to demand shocks than productivity shocks. It is plausible that the recent shift in demand and rise in productivity have caused the yuan to be undervalued, and further shocks in the future will necessitate more adjustment. In addition, by introducing the labour market, we endogenise the rise in the real wage. We show that consumption in China will grow faster than the real wage because China is a creditor country and receives debt interest payment from the rest of the world.

The paper is organised as follows. In section 2 we review the background and debates of the Chinese renminbi undervaluation. Section 3 sets up a simple two-country DSGE model in which country assets are imperfect substitutes. Section 4 simulates the model and discusses model implications. Section 5 concludes and presents some suggestions for further research.

2 Background

2.1 The Exchange Rate Regime in China

Following McKinnon et al. (2008), the evolution of China's exchange rate regime can be broadly categorised into three phases: currency inconvertibility before 1994, the fixed dollar exchange rate from 1995 to 21 June 2005, and a predictable upward crawl through today.

Before 1994, the Chinese renminbi was inconvertible. There were multiple exchange rates and exchange controls on both current and capital account transactions. Without capital mobility and arbitrage opportunity between China and the rest of the world, how the exchange rate was set was arbitrary. The official rate was not economically very meaningful.

In 1994, the Chinese authorities abolished exchange controls on current account transactions and unified the exchange rate. By 1995, the nominal exchange rate had settled down to 8.28 yuan per dollar. The rate was fixed at this point with a narrow band of 0.3 percent for 10 years. The main motivation was to stabilise the price level. Since 1996, the current account has been fully convertible.

After 2003, increasing current account surplus, together with large inflows of FDI, led to a large buildup of balance of payment surplus. Also, inflow of speculative money and political pressure from America stressed the old system.

On July 21, 2005, China revalued the yuan exchange rate against the U.S dollar by 2.1 percent, and switched from the dollar peg to a basket with undisclosed proportions. A daily fluctuation band against the dollar is set to be less than 0.3%, subsequently widened to 0.5%. In August 2005, The People's Bank of China (PBoC) specified that the basket includes the dollar, the euro, and the yen and the Korean won but also the Malaysian ringget, the Russian ruble, the Australian dollar, the Thai baht, the Canadian dollar and the British pound.

Some researchers estimate the basket weights by regressing the log first-differenced exchange rate of the Chinese renminbi against other major currencies. An early estimate by Eichengreen (2006) using data from July 2005 to March 2006 find that the weight on the dollar is on the order of 0.9, while all other currencies is not statistically significant. Frankel and Wei (2007) estimate from sub samples and report that the dollar weight falls to 70% at one point in February-April 2006, while the Malaysian ringgit, the Korean won, the Russian ruble and the Thai baht receive positive weights. A renewed estimation by Frankel (2009) accounting for the flexibility of the basket and trend appreciation, reveals that by mid-2007 the renminbi basket had switched a substantial part of the dollar's weight onto the euro (40%).

2.2 Equilibrium Exchange Rate of the Chinese Yuan

There are heated debates on the existence and extent of undervaluation of the Chinese renminbi in the academic and political spheres alike.[4] Cline and Williamson (2008) review and discuss the various recent empirical estimation of the equilibrium exchange rate of the Chinese yuan.

Three approaches are widely used to study the equilibrium exchange. They are the enhanced-PPP approach, behavioral equilibrium exchange rate (BEER) approach and the fundamental equilibrium exchange rate (FEER) approach. The PPP approach refers to finding the equilibrium exchange rate that leads to absolute PPP. Using multiple country data, the real exchange rate is regressed against real per capita income to correct for the

[4]For instance, Timothy F. Geithner, then the nominee for Treasury secretary of the United States, wrote in a written testimony to the Senate: 'President Obama – backed by the conclusions of a broad range of economists – believes that China is manipulating its currency.' (Jan 23 2009)

Balassa-Samuelson (1964) effect. Deviation from the regression line indicates exchange rate misalignment of a particular country. However, the enhanced-PPP approach does not consider other real factors potentially affecting the real exchange rate. These factors include net foreign assets, terms of trade, trade openness and government spending. The BEER approach accounts for these real factors and decompose total misalignment into deviations of economic fundamentals from their sustainable values, transitive effects and random disturbances. The FEER approach, sometimes known as macroeconomic balance, searches for an equilibrium exchange rate that achieves internal balance and external balance at the same time. The internal balance can be defined as the combinations of demand and the real exchange rate that lead to full employment and price stability. The external balance refers to the equilibrium in the current account. The FEER is thus the real exchange rate at which the two schedules intersect.

Most of the estimates using each of the three approaches indicate significant yuan undervaluation, both in real effective exchange rate (REER) and bilateral U.S dollar rate. For instance, Frankel (2006) and Cheung, Chinn and Fujii (2007) use the enhanced-PPP approach and find that the Yuan is 45% and 69% undervaluated against the U.S dollar respectively.[5] Benassy-Quere et al. (2004) and Benassy-Quere et al. (2006) employ the BEER approach and report 34-36% and 26-46% undervaluation respectively in the bilateral exchange rate. FEER estimations by Wren-Lewis (2004) and Cline (2007) give 17-20% and 29-33% yuan undervaluation against the U.S dollar respectively.[6]

We do not claim that these approaches are perfect. In fact, each approach has its own limitations. PPP ignores other real economic factors and assumes that the U.S dollar cannot be over- or undervalued. In BEER, the sustainable values of the fundamentals are not clear. FEER estimates are sensitive to the large transitory fluctuations of the current account balance. Despite the limitations, recent studies seem to support the claim that the Chinese real exchange rate is undervalued.[7]

2.3 A Suitable Exchange Rate Regime for China

Many authors have discussed the advantages of fixed and floating exchange rates. Commonly used arguments for fixing include: providing a nominal anchor to monetary policy, encouraging trade and investment, precluding competitive depreciation, and avoiding speculative bubbles. The four corresponding advantages for floating are: giving independence to monetary policy, allowing automatic adjustment to trade shocks, retaining seigniorage and lender-or-last-resort capability, and avoiding speculative attacks (Frankel, 2003).

Given the large territory and population size in China, one would expect that some

[5] All these percentages are reported in logarithmic terms.

[6] For a complete summary, see Cline and Williamson (2008).

[7] The word 'undervaluation' or 'overvaluation' has no single agreed upon definition among economists. Nevertheless, the renminbi seems now to qualify for most of the various possible definitions of undervalued.

internal shocks (eg. earthquakes) are not highly correlated with external shocks. A flexible exchange rate is likely to be beneficial, as having the freedom to implement monetary policy helps stabilise the economy without excessive price adjustment. Consider, for example, a negative demand shock specific to the domestic economy, under fixed exchange rate, the authority may either expand fiscally, which is less efficient, or let price level adjust slowly downwards, making a recession long and painful. Alternatively, if the demand shock happens in the foreign economy to which the domestic currency is pegged, the foreign interest rate will fall, and, under a fixed exchange rate arrangement, the domestic interest rate has to follow, which leads to overheating.

The fixed exchange rate fared relatively well in China in 1994-2005, when China maintained annual GDP growth of about 10% with negligible inflation. However, the growth during this period has been led by exports and investments. When the government contemplates for a more balanced growth and stimulates internal consumption, the role of monetary independence will become more crucial. In particular, there is a need now to better tailor money and credit conditions to the state of the Chinese economy. In 2008, for example, when there is evidence of overheating in China, easy monetary policy in the U.S was not helpful. There would seem to be a case for a tighter monetary policy rather than one that is dictated by foreign conditions and the currency peg.

Although fixed exchange rate reduces exchange rate uncertainty, renminbi can only be pegged with one other currency. However, China's trade pattern is too diversified to peg to any single major currency. China's major export destinations include the U.S, Japan and the European Union, while top importers come from Japan and ASEAN countries (Goldstein and Lardy 2008). An obvious solution is a basket peg instead. However, although a basket peg reduces currency fluctuation, it does not eliminate it. A basket peg is also less firm and credible than pegging with a single currency, especially when weights are not announced. It is possible that the trade promotion effect is considerably smaller than a firm fix. In addition, under a basket peg, monetary policy will not be free. Instead, it will be a trade-weighed average of the policy rates of the currencies the basket contains.

In terms of empirical evidence, Rogoff et al. (2003) use annual data from 1970 to 1990 for up to 158 countries and confirm that while countries that fixed exchange rate regimes appear to offer some measure of credibility without compromising growth objectives, as countries develop economically and institutionally, there appear to be considerable benefits to adopting a more flexible exchange rate system, although this finding should not be regarded as a one-size-fit-all prescription. Starting from a closed and developing economy, China has sustained a double-digit growth for more than twenty years. Despite a variety of institutional weaknesses, fragile banking system and other sources of macroeconomic volatility, the value of exchange rate flexibility is likely to be increasing.

We do not argue that the floating exchange rate is the only regime that is suitable for China. There are many intermediate regimes between a hard peg and a pure float, such as a crawling peg and an adjustable peg. The bipolar view does not rule out all the intermediate regimes (Fischer, 2001). Joshi (2003) suggests intermediate regimes may be suitable for emerging markets. We only argue that more flexibility of the renminbi is beneficial to the Chinese economy.

2.4 Exiting the Fixed Exchange Rate Regime

If more currency flexibility is desirable in China, an orderly exit strategy is essential. Proposals by economists are broadly classified into three categories. The first proposal is a move towards a managed flow. The second involves an immediate step revaluation and a subsequent shift to a more flexible exchange rate. The third is the case for the status quo: returning to a fixed rate until other internal imbalances are settled.

2.4.1 A Gradual Move Towards Flexibility

Under this proposal,[8] although the currency is allowed to fluctuate, the government will still manage the exchange rate heavily. There will be government interventions but no exchange rate targets. The PBoC will have to 'lean against the wind': to increase the supply of renminbi when appreciation is excessive and sell off foreign assets if there is a pressure of a large deprecation. With two trillion U.S dollars in the foreign reserves, the PBoC will not find it difficult to manage the exchange rate in the presence of some forms of capital controls.

Increasing currency flexibility will allow a larger degree of monetary independence. As we argued, it is advantageous for China to have some degree of autonomy in monetary policies. Flexibility of the currency breaks the impossible trinity and allows the central bank more freedom to use monetary policies to deal with domestic shocks.

However, abandoning the peg also means abandoning the existing normal anchor. A new anchor is needed to contain inflation. Eichengreen (2004) and Goodfriend et al. (2007) suggests a rudimentary inflation-targeting type anchor. If this is the case, the credibility of the central bank is important, for the government may be tempted to run inflation-biased policies to overheat the economy. That implies the need of a more independent central bank in China. The reputation takes time to build up.

Others doubt whether inflation targeting is suitable for China at this stage. McKinnon et al. (2008) point out that the absence of a well-developed domestic bond market and the fragmented structure of interest rates (accompanied by differentiated direct credit controls in various lending categories) make the nominal interest an unsatisfactory instrument to use in the Taylor rule. Moreover, the authorities in China may not have timely and accurate statistical information about the output gap due to high and somewhat unpredictable real economic growth. Goodfriend et al. (2007) argue that monetary policy credibly geared towards targeting low inflation must be accompanied by a willingness on the part of the government and the public to allow a substantial degree of flexibility in the

[8] See Eichengreen (2004, 2006), Prasad (2007) for example.

foreign exchange rate. The PBoC may not be able to fully commit itself to using interest rates for inflation objectives, as it may trigger hot money inflows and reduce state bank profitability.

Also in this proposal of increasing flexibility is a larger volatility of the exchange rates so that the currency can appreciate as well as depreciate in the short run. Eichengreen (2006) suggests that the daily fluctuation band against the U.S dollars should be eliminated because the existence of a two-way bet will deter speculators from all lining up on one side of the market.

While Eichengreen (2006) points out that an appreciation expectation associated with currency flexibility will lead to worrisome capital inflows and exasperate the risk of domestic inflation and asset price bubbles, it is not entirely clear why more fluctuation can create a two-way bet for the speculators. The real exchange rate is undervalued. To ensure smooth exchange rate realignment, part of the adjustment will be through nominal exchange rate appreciation. Along the adjustment path, the nominal exchange rate is still a one-way bet on the appreciation side. Creating excessive volatility may only impede speculative inflows of short horizon. For flows with a longer horizon the one-way bet does not disappear.

2.4.2 Step Revaluation with a Move Towards Flexibility

The revaluation and flow is the essence of Goldstein and Lardy's (2003, 2008) two-step adjustment proposal. The argument is that either a step revaluation or a gradual move towards exchange rate flexibility alone is not desirable. Here is why:

A step revaluation will not enhance the authorities' ability to tailor financial and foreign-exchange market conditions to domestic circumstances. Monetary policy will still be constrained. When domestic prices gradually adjust to the new level of nominal exchange rate, the problem of real exchange rate undervaluation will reappear. The private sector will expect another step revaluation because it had happened before. Repeated step revaluation will undermine the central bank's credibility, encouraging speculative capital inflow without solving the original exchange rate problem. Moreover, there is no agreement as to how large a jump it would take to restore external equilibrium. Different approaches (documented in Section 2.2) yield different answers. A one-time appreciation that is too small will induce expectations for further jumps and trigger hot capital inflow which necessitates further revaluations.

On the other hand, Frankel (2006) argues that a step revaluation is important in the two-step proposal. According to his calculations, if the Chinese real exchange rate undervaluation is to be closed by half in ten years' time and if China maintains a GDP growth 6 percent higher than that of the U.S, a real exchange rate appreciation of 4.5% is necessary. Assuming perfect capital mobility, the uncovered interest parity (UIP) would imply that real interest rate in China will have to be 4.5% below U.S interest rate for ten years, which is undesirable. A one-off revaluation may bring down the future interest differential. In addition, a smaller appreciation may be expected in the future, which reduces the incentive for one-way bet and speculative inflow into China.

The question then is: how large the initial appreciation should be. A substantial step appreciation might be a policy mistake for the following reasons. First, exports and FDI will drop. These have direct impacts to growth and unemployment since both are major drivers of the Chinese economy. The underdeveloped social security system makes matters worse. Second, firms may suffer from losses associated with unhedged foreign exposures. While banks and large multinational firms are somewhat insulated,[9] smaller local private firms will be hit hard. Third, collapsing firms will increase non-performing loans (NPLs) in banks. Anderson (2006a) argues that NPLs together with a closing of lending-deposit spread (another ongoing reform in Chinese banks) can easily erase state bank profits. These are possible consequences of too large an initial revaluation.

The above suggests that the initial appreciation can be neither too big nor too mild. A jump too mild may lead to over-investment and overheating; on the contrary, an excessive revaluation can hurt growth and employment. The government has an active role to play. An expansionary fiscal policy helps to strengthen domestic demand to compensate for the fall in exports. Reorienting some government spending towards essential social expenditure, including health care and education, and reducing uncertainty on these fronts may give households confidence to increase consumption levels (Blanchard and Giavazzi, 2005). The low levels of explicit government deficits and public debt provide some room for maneuver in these areas (Prasad and Rajan, 2006).

2.4.3 Return to Firm Fix

Proponents of a fixed rate argue that great flexibility in renminbi attracts speculative hot money inflow and as a result the central bank is losing monetary control rather than gaining it.[10] They contend that the right way to reduce trade surplus is to reduce savings, and the government should cut tax, increase social expenditures and formalise dividend payouts. The inflationary impact has to be contained by monetary tightening. PBoC needs exchange rate stabilisation to regain monetary control, because a credible hard peg gives speculators no incentive to bet against the yuan. The precise level of exchange rate is a less important concern.

In particular, McKinnon (2007) and McKinnon and Schnabl (2008) maintain that the exchange rate cannot affect the trade balance. Even if the Marshall-Lerner condition is satisfied, the substitution effect is offset by an income effect in at least three channels. Suppose there is an exchange rate appreciation. First, the fall in exports itself tends to reduce domestic national income and spending, including spending on imported goods. Second, the exchange rate makes new investment projects look more expensive to foreign

[9]See J.P. Morgan Chase, *Global Data Watch*, (February 4, 2005) for details.
[10]See Mundell (2004), McKinnon(2007) and McKinnon and Schnabl (2008) for example.

investors, which implies that FDI will fall, causing a further fall in domestic expenditures including imports. Third, in creditor countries that have built up large dollar claims on foreigners, an exchange appreciation also has a negative effect. A recent empirical study on Chinese trade elasticity by Cheung et al. (2009) estimated that a 10% appreciation in bilateral rate would *increase* trade balance by 4%, because Chinese import also declines. Firm-fix proponents fear that exchange rate will have to keep rising without closing the trade surplus, resembling Japan-bashing story in the eighties.

However, nothing precludes an appreciation together with a fiscal expansion, and in which case the negative income effect is compensated by an increment in fiscal expenditure and internal demand. As the real exchange rate rises, trade surplus falls accordingly. A combination of bolder policies is required for a smooth transition, as Blanchard and Giavazzi (2005) and Prasad and Rajan (2006) explain.

The major concern of the hard-peg proponents is the overwhelming speculative flows in anticipation of currency appreciation. During exchange rate regime transition, the flow can be temporarily dealt with by sterilisation and capital controls. While sterilisation creates stress towards the central bank reserves and therefore cannot last forever, the PBoC has a very strong balance sheet in the short run. Anderson (2006a) maintains that the scope of Chinese sterilisation operations is still quite mild, both as a share of base money and of GDP, by regional standards. Sterilisation has its limits, so capital controls is another strategy to deter unwanted capital flows. Joshi (2003) argues that an intermediate regime with capital controls is a feasible approach to the impossible trinity in developing countries, and that capital controls have been effective in many cases to deter some speculative flows.

2.5 Relation to Other Reforms

The issue of renminbi appreciation is inter-connected with other imbalances in China. These imbalances are low consumption (43%), high investment (42%) and exports (31%) to GDP ratio, financial sector and banking sector distortions and the existence of capital controls, to name a few.[11] Yu (2008) documents the history and current features of capital control in China. He argues that the many distortions we see now are the results of a policy choice by the Chinese government, namely a fixed exchange rate regime, capital control with preferential policy towards FDI and controlling state banks as a government branch of credit rationing.

To facilitate the change in exchange rate regime, other policies or reforms, such as fiscal expansion (Blanchard and Giavazzi 2005, Prasad and Rajan 2006), liberalisation of capital account (Goldstein et al. 2008, Yu 2008),[12] financial and banking sector reform

[11] Source: Blanchard and Giavazzi (2005).

[12] Capital account has to be liberalised cautiously (Fischer 2001). When the exchange rate is fixed and banking sector weak, sudden large reversals of capital flow may damage the financial system. Eichengreen (2004) contends that the move to significantly greater exchange rate flexibility should precede further

(Prasad and Rajan 2006, Goodfriend et al. 2007, Yu 2008), should move along in tandem and with coordination. For example, the banking sector reform would require the PBoC to avoid using bank reserve requirement to fight inflation. However, raising the policy rate will attract hot money as the capital account liberalises. Rapid exchange rate appreciation would reduce growth and fiscal spending would have to expand. The buildup of contingent liabilities in banks could put a bind on fiscal policy. The example illustrates the complexity of the reform and the need of extensive policy coordination.

The discussion above motivates us to build a model that incorporates the real economy and imperfect capital mobility, which allows us to understand the relations between a real exchange rate adjustment and other real variables in the system. In particular, we will show firstly that a shift in consumers' taste towards Chinese goods may have caused the Chinese renminbi to be undervalued; and secondly, a relative increase in productivity in China moderates the need for the currency to appreciate along the adjustment path. However, as we will explain, the first effect is likely to dominate, giving rise to a step jump and a subsequent gradual appreciation in the real exchange rate. The adjustment corresponds to the Goldstein and Lardy (2003, 2008) two-step revaluation proposal and allows us to analyse their arguments systematically under the assumptions of our model.

3 A Model with Imperfect Capital Mobility

3.1 Background

Many theoretical international macroeconomic models assume the uncovered interest parity (UIP) condition in their models (e.g Clarida, Gali and Gertler 2001, Gali and Monacelli 2002). However, empirical studies have found large and sustained deviations from the UIP (Sarno and Taylor, 2003). In the context of U.S and Chinese financial asset trade, since cross-border capital flow is monitored by the Chinese government, the UIP may not hold in the short run.[13] However, over time, capital controls tend to lose their effectiveness and efficiency (Fischer 2001, Eichengreen 2004, Goodfriend et al. 2007).[14]

Blanchard, Giavazzi and Sa (2005) first use a gross rate of return formulation to investigate the U.S dollar exchange rate and U.S current account deficit. The gross rate of return on holding U.S assets versus foreign assets, R, is given by:

$$R \equiv \frac{1+r}{1+r^*}\frac{E'^e}{E}$$

capital account liberalisation. While significant capital account restrictions rule out a free float, they do not diminish the feasibility or desirability of a managed float.

[13] It is likely that there are imperfect mobility due to non-policy barriers, such as risk premium. The proposed model may have wider applications.

[14] There are illegitimate ways to get around the capital control. In China, for example, it is not uncommon to over-invoice exports, under-invoice imports, exchange in black markets and disguise speculative money flows under the name FDI to direct funds into the country.

where r is the U.S interest rate, r^* is the foreign interest rate, E denotes the exchange rate, defined as the price of U.S goods in terms of foreign goods, and $E^{'e}$ denotes the expected exchange rate in the next period. Therefore, the gross rate of return on foreign assets in terms of U.S goods is $(1 + r^*)E/E^{'e}$. If the UIP holds, $R \equiv 1$. In their model R may be different from one in the short run.

Kirsanova, Kuralbayeva and Vines (2008) (henceforth KKV) adopt the formulation and adapt it into a DSGE model. Our model is created in the spirit of KKV (2008), and we expand it in a number of ways. First, we introduce labour in the utility function. Households own firms which produce output using labour as a factor of production. Second, we introduce product differentiation with infinite number of goods. Monopolistically competitive firms earn positive profits as a result. Third, we assume that firms are profit-maximising, but subject to a Calvo (1983) type price setting rule. Fourth, we introduce the monetary authorities in the flexible exchange rate environment. It sets nominal interest rate according to a Taylor rule. Fifth, we link the demand shock to a shift in home consumption preferences.

This model enables us to analyse the behavior of the economy around the equilibrium when it faces a productivity shock. The assumption of imperfect capital mobility between domestic and foreign assets is crucial to the saddle-path adjustment of the solution.

3.2 The Model

3.2.1 Consumers

The model is a two country model with forward looking consumers. Individuals have same preferences in both countries.

$$U = \mathscr{E}_0 \sum_{t=0}^{\infty} \beta^t (\ln C_{Jt} + \ln G_{Jt} - \frac{\nu}{1+\varphi} L_{Jt}^{1+\varphi}) \tag{1}$$

where C aggregate consumption, G is government spending, L is labour and μ_t is a time-varying consumption home bias. $\mathscr{E}$ is the expectation operator. ν is a constant weight. φ is the inverse of labour supply elasticity. The two countries are labeled as $J \in \{A, B\}$. Consumers purchase both home and foreign goods:

$$C_{Jt} = \left(\mu_{Jt}^{\frac{1}{\eta}} C_{JHt}^{\frac{\eta-1}{\eta}} + (1 - \mu_{Jt})^{\frac{1}{\eta}} C_{JFt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{2}$$

and price index P_{Jt} is defined as follows:

$$P_{Jt} = \left(\mu_{Jt} P_{JHt}^{1-\eta} + (1 - \mu_{Jt}) P_{JFt}^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{3}$$

The parameter $\eta > 1$ measures the elasticity of substitution between domestic and foreign goods. The demand of each country's good is given by:

$$C_{JHt} = \mu_{Jt} (\frac{P_{JHt}}{P_{Jt}})^{-\eta} C_{Jt}, \qquad C_{JFt} = (1 - \mu_{Jt}) (\frac{P_{JFt}}{P_{Jt}})^{-\eta} C_{Jt} \tag{4}$$

In each country, firms produce a continuum of infinite goods in a monopolistically competitive way:

$$C_{JHt} = \left(\int_0^1 C_{JHt}(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}, \qquad P_{JHt} = \left(\int_0^1 P_{JHt}(z)^{1-\epsilon} dz \right)^{\frac{1}{1-\epsilon}}$$

and the demand schedule is:

$$C_{JHt}(z) = (\frac{P_{JHt(z)}}{P_{JHt}})^{-\epsilon} C_{JHt} \tag{5}$$

An individual chooses optimal consumption and work effort to maximise the utility function (1) subject to the demand system and the budget constraint:

$$A_{Jt+1} = (1 + \rho_{Jt})[A_{Jt} + (1 - \tau)(w_{Jt} L_{Jt} + \Pi_{Jt}) - P_{Jt} C_{Jt}] \tag{6}$$

where $P_{Jt}C_{Jt}$ is aggregate nominal consumption, A_{Jt} are nominal financial assets of a household, w are nominal wages, τ is an exogenous income tax rate and Π are profits. The household's wealth accumulation must satisfy the no Ponzi game condition. Assume $M^J_{t,t+1}$ be the stochastic discount factor which determines the price in period t to the individual of being able to carry a state-contingent account A_{Jt+1} of wealth into period $t+1$, then on average it has to equal the discount rate implied by the return on financial wealth ρ_{Jt}:

$$\mathscr{E}_t(M^J_{t,t+1}) = \frac{1}{1 + \rho_{Jt}}, \qquad \text{where} \quad M^J_{t,t+i} = \beta \frac{P_{Jt} C_{Jt}}{P_{Jt+i} C_{Jt+i}}$$

The no Ponzi game condition, $\lim_{s \to \infty} \mathscr{E}_t(M^J_{t,s} A_{Js}) = 0$, is the terminal condition of the dynamic optimisation problem, without which consumers can borrow infinitely. Household optimisation leads to the following first order conditions:

$$\beta(1 + \rho_{Jt}) E_t \left(\frac{P_{Jt} C_{Jt}}{P_{Jt+1} C_{Jt+1}} \right) = 1 \tag{7}$$

$$\nu L^{\varphi}_{Jt} C_{Jt} = (1 - \tau) \frac{w_{Jt}}{P_{Jt}} \tag{8}$$

The first equation is just the conventional Euler equation which shows the intertemporal trade-off of consumption. The second equation is the intratemporal optimal condition equating marginal utility of consumption and marginal disutility of labour.

3.2.2 Exchange Rates

We assume the law of one price holds. The nominal exchange rate E_t is given by

$$E_t = \frac{P_{AHt}}{P_{BFt}} = \frac{P_{AFt}}{P_{BHt}} \tag{9}$$

We also define the real exchange rate, S_{At}, as the price of import in country A to the price of home goods in country A.[15]

$$S_{At} = \frac{P_{AFt}}{P_{AHt}} = \frac{P_{BHt}}{P_{BFt}} \tag{10}$$

An increase in S_{At} corresponds to a real depreciation of the exchange rate. $S_{Bt} = 1/S_{At}$ as the real exchange rate in the perspectives of the foreign country.

3.2.3 Firms and Price-setting

A typical firm produces a differentiated good with linear technology represented by the production function

$$Y_{Jt}(z) = \mathscr{A}_{Jt} L_{Jt}(z) \tag{11}$$

where $\mathscr{A}$ is a country-specific stochastic productivity shock with mean one. Hence, the real marginal cost will be common across firms in each country. It is useful to derive an aggregate production function relating aggregate output and aggregate employment. We define

$$L_{Jt} \equiv \int_0^1 L_{Jt}(z)dz = \frac{Y_{Jt}\int_0^1 \frac{Y_{Jt(z)}}{Y_{Jt}}dz}{\mathscr{A}_{Jt}}$$

Following the appendix of Gali and Monacelli (2005) and Gali (2008), we can show that the integral ${Y_{Jt}}^{-1}\int_0^1 Y_{Jt(z)}dz$ is of second order. Thus, up to a first-order approximation, the log aggregate production function is

$$\hat{Y}_{Jt} = \hat{\mathscr{A}}_{Jt} + \hat{L}_{Jt} \tag{12}$$

where the 'hat' sign denotes the log-deviation from the steady state.

Next, we consider firms' price-setting. Firms maximise profits and use the price setting model due to Calvo (1983). Each period firms that adjust their prices are randomly selected. A fraction (1-κ) adjusts while remaining does not adjust. The firms' pricing decision problem then involves picking $P_{AHt}(z)$ to maximise expected discounted future profits. The price in the foreign market is $P_{BFt}(z) = P_{AHt}(z)E_t$, due to the law of one price. This is the only nominal rigidity in the model.

The maximisation problem faced by country A firms is:

$$\max_{P_{AHt}(z)} \mathscr{E}_t \sum_{i=0}^{\infty} \kappa^i M^A_{t,t+i} \left(P_{AH}(z) - MC^n_{At+i}\right) Y_{At+i}(z) \tag{13}$$

[15] Strictly speaking, S_t is the terms of trade, as it is the ratio of the price of imported goods to the price of domestic goods. Since we do not define separately a 'real' real exchange rate $E_t \times (P_{Bt}/P_{At})$, and the log-deviation of real exchange rate from the steady state is an affine transformation of that of the terms of trade, we use the two terms interchangeably.

where $MC^n_{At} = \frac{(1-\tau)w_{At}}{\mathscr{A}_{At}}$ denotes the nominal marginal cost. By symmetry, every firm that can adjust price will set the same price. Denote this price as $\tilde{P}_{AHt}$. The first order condition is given by:

$$\mathscr{E}_t \sum_{i=0}^{\infty} (\beta\kappa)^i \left((1-\epsilon) \frac{\tilde{P}^{-\epsilon}_{AHt}}{P^{1-\epsilon}_{AHt+i}} + \epsilon \frac{\tilde{P}^{-\epsilon-1}_{AHt}}{P^{-\epsilon}_{AHt+i}} MC_{At+i} \right) \frac{Y_{At+i}}{C_{At+i}} = 0$$

$$\therefore \quad \tilde{P}_{AHt} = \frac{\epsilon}{\epsilon - 1} \frac{\mathscr{E}_t \sum_{i=0}^{\infty} (\beta\kappa)^i P^{\epsilon}_{AHt+i} MC_{At+i} \frac{Y_{At+i}}{C_{At+i}}}{\mathscr{E}_t \sum_{i=0}^{\infty} (\beta\kappa)^i P^{\epsilon-1}_{AHt+i} \frac{Y_{At+i}}{C_{At+i}}} \tag{14}$$

where MC is the real marginal cost

$$MC_{At+i} = \frac{(1-\tau)w_{At+i}}{P_{AHt+i}\mathscr{A}_{At+i}} = \frac{MC^n_{At+i}}{P_{AHt+i}} \tag{15}$$

The average period t price satisfies

$$P_{AHt} = \left((1-\kappa)\tilde{P}^{1-\epsilon}_{AHt} + \kappa P^{1-\epsilon}_{AHt-1} \right)^{\frac{1}{1-\epsilon}} \tag{16}$$

Equations (14) and (16) together produce the standard Calvo pricing Phillips Curve.

$$\hat{\pi}_{At} = \frac{1}{\lambda} \hat{MC}_{At} + \beta \mathscr{E}_t \hat{\pi}_{At+1} \tag{17}$$

where λ is defined as $\kappa/[(1-\kappa)(1-\kappa\beta)]$ and $\hat{\pi}_{At} = \ln(P_{AHt}/P_{AHt-1})$. The derivation of this Phillips curve is given in the Appendix A.1.

3.2.4 Government

The government in country J buys only domestic goods (G_{Jt}), and its allocation across goods is determined by minimising total costs.

$$G_{Jt} = \left(\int_0^1 G_{Jt}(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}, \qquad G_{Jt}(z) = (\frac{P_{JHt(z)}}{P_{JHt}})^{-\epsilon} G_{Jt}$$

It also taxes income (with tax rate τ) and issues nominal debt B_{Jt}. The evolution of the nominal debt stock can be written as:

$$B_{Jt+1} = (1 + i_{Jt})(B_{Jt} + P_{JHt}G_{Jt} - \tau P_{JHt}Y_{Jt}) \tag{18}$$

where i_{Jt} is nominal interest rate on debt in country J.

Government debts are the only tradable assets across the countries. The supply of the government assets is limited by the amount of government debt and dependent on government purchases and income tax revenues. This allows us to analyse the effects of China running a temporary current account surplus, which influence the evolution of the debts and supply of assets.

3.2.5 Financial Markets

Following KKV (2008), we assume the two debts are not perfect substitutes and the expected excess return for country A, Z_{At}, is defined as:

$$Z_{At} = \frac{(1 + i_{At})E_t}{(1 + i_{Bt})E^e_{t+1}} \tag{19}$$

This formulation of the expected excess return compares the investment return of country A bonds with investment return of country B bonds adjusted by expected nominal exchange rate change from period t to $t+1$. Similarly, from the perspective of country B consumers, their excess investment return is $Z_{Bt} = 1/Z_{At}$. If the UIP holds $Z_{Jt} \equiv 1$ for all t.

Households in each country prefer keeping their country debt if returns are the same, i.e. there is a home asset bias. Let α_{Jt} be a proportion of financial wealth A_{Jt} that is invested in bonds B_{Jt}. It follows that

$$\alpha_{At}A_{At} + (1 - \alpha_{Bt})A_{Bt}E_t = B_{At} \tag{20}$$

$$(1 - \alpha_{At})A_{At} + \alpha_{Bt}A_{Bt}E_t = B_{Bt}E_t \tag{21}$$

and the return on wealth is

$$1 + \rho_{At} = (1 + i_{At})\left(\alpha_{At} + \frac{(1 - \alpha_{At})}{Z_{At}}\right) \tag{22}$$

$$1 + \rho_{Bt} = (1 + i_{Bt})\left(\alpha_{Bt} + (1 - \alpha_{Bt})Z_{At}\right) \tag{23}$$

Finally, we assume simple relationships of the following form:

$$\alpha_{At} = \alpha Z^{\phi}_{At}, \qquad \alpha_{Bt} = \alpha Z^{-\phi}_{At} \tag{24}$$

where $\alpha > 0.5$ is the steady state home asset bias, the parameter ϕ is interpreted as a measure of the responsiveness of capital flows to changes in excess return. For instance, when ϕ tends to infinity, capital is perfectly mobile and the UIP holds. For small ϕ, the country asset holding is not sensitive to changes in excess return.

3.2.6 Monetary Authorities

Monetary authorities follow a Taylor-rule type policy. Country J's monetary rule is represented by

$$(1 + i_{Jt}) = \beta^{-1}\left(\frac{P_{JHt}}{P_{JHt-1}}\right)^{\gamma} \tag{25}$$

The monetary authorities use a rudimentary type of Taylor rule which does not take into account any change in output gaps directly. While the latter rule introduces 'derivative' control of inflation, it makes the model less tractable. We assume that the monetary authorities work independently from the government (no fiscal dominance). Nominal interest rate responds to current account surplus and deficits only to the extent that current account adjustment affects the inflation rate.

3.2.7 Aggregate Demand

Aggregate demand comprises consumption demand coming from domestic residents, consumption demand from foreign residents and government consumption which is directed towards domestical goods only:

$$Y_{At} = \mu_{At}(\frac{P_{AHt}}{P_{At}})^{-\eta}C_{At} + (1-\mu_{Bt})(\frac{P_{BFt}}{P_{Bt}})^{-\eta}C_{Bt} + G_{At} \quad (26)$$

$$Y_{At}(z) = (\frac{P_{AHt(z)}}{P_{AHt}})^{-\epsilon}Y_{At} \quad (27)$$

With the aggregate demand, we can compute firm profits in each country:

$$\begin{aligned}\Pi_{At} &= \mu_{At}(\frac{P_{AHt}}{P_{At}})^{-\eta}C_{At}P_{AHt} + (1-\mu_{Bt})(\frac{P_{BFt}}{P_{Bt}})^{-\eta}C_{Bt}\frac{P_{BFt}}{E_t} + G_{At}P_{AHt} - w_{At}L_{At} \\ &= P_{AHt}Y_{At} - w_{At}L_{At} \quad (28)\end{aligned}$$

Using this result, the budget constraint (6) can be re-written as the following:

$$A_{At+1} = (1+\rho_{At})(A_{At} + (1-\tau)P_{AHt}Y_{At} - P_{At}C_{At}) \quad (29)$$

3.3 Steady State and Log-linearisation

3.3.1 The Steady State

We assume a symmetric steady state and unit equilibrium exchange rate for simplicity. Then the steady state is straight-forward:

$$\begin{aligned}&Y_A = Y_B \\ &C_A = C_B \\ &P_{AH} = P_{AF} = P_A = P_B = P_{BH} = P_{BF} \\ &E_A = E_B = S_A = S_B = 1 \\ &A_A = A_B = B_A = B_B \\ &\mu_A = \mu_B = \mu\end{aligned}$$

Furthermore, we denote the steady state ratio of stock of domestic bonds to output (B_A/Y_A) as B. We also assume that the share of government in the economy $(G_J/Y_J) = (1-\theta)$ so that share of consumption in the economy is $(C_J/Y_J) = \theta$.

We log-linearise around the symmetrical steady state so that for each variable X_{Jt}, the log-deviation is[16]

$$\hat{X}_{Jt} = \ln X_{Jt} - \ln X_J = \ln(\frac{X_{Jt} - X_J}{X_J} + 1) \approx \frac{X_{Jt} - X_J}{X_J}$$

[16]This is with the exception of $x = \{r, \rho, i\}$, where we define $\hat{x}_{Jt} = (x_{Jt} - x)/(1 + x_J)$.

We use the following first-order approximation rule $X_{Jt} \approx X_J(1+\hat{X}_{Jt}) + O(\epsilon^2)$.

In the following, we define

$$\tilde{A}_{Jt} = \frac{A_{Jt}}{P_{JHt}} \qquad \tilde{B}_{Jt} = \frac{B_{Jt}}{P_{JHt}} \tag{30}$$

and $\hat{A}$ and $\hat{B}$ are the log-deviation of the $\tilde{A}$ and $\tilde{B}$ variables. For convenience, we also define the net debt variable $\hat{F}$:

$$\hat{F}_{At} = \hat{B}_{At} - \hat{A}_{At} \tag{31}$$

$$\hat{F}_{Bt} = \hat{B}_{Bt} - \hat{A}_{Bt} = -\hat{B}_{At} + \hat{A}_{At} = -\hat{F}_{At} \tag{32}$$

Finally, we define the lower case letter x_t for each variable be the relative log-deviation from the steady state between the two countries, that is

$$x_t = \hat{X}_{At} - \hat{X}_{Bt}$$

The technical details of the log-linearisation are given in the Appendix A.2.

3.3.2 The Log-linearised Complete model

After the log-linearisation, we obtain the following complete system in differences:[17]

$$z_t = 2r_t + s_t - s^e_{t+1} \tag{33}$$

$$2\alpha\phi z_t = 2(1-\alpha)b_t + (2\alpha-1)f_t - (1-\alpha)s_t \tag{34}$$

$$y_t = (2\mu-1)\theta c_t + 2\mu(1-\mu)\theta\eta s_t + (1-\theta)g_t + \varsigma_t \tag{35}$$

$$c_t = c^e_{t+1} - r_t + (1-\alpha)z_t + (1-\mu)(s^e_{t+1} - s_t) \tag{36}$$

$$b_{t+1} = r_t + \frac{1}{\beta}\left(b_t + \frac{(1-\theta)}{B}g_t - \frac{\tau}{B}y_t\right) \tag{37}$$

$$f_{t+1} = (1-\alpha)z_t + \frac{1}{\beta}\left(f_t + 2\frac{\theta}{B}(1-\mu)c_t - \frac{(1-\mu)\theta}{B}(2\mu\eta-1)s_t - \frac{1}{B}\varsigma_t\right) \tag{38}$$

$$r_t = \gamma\pi_t - \pi^e_{t+1} \tag{39}$$

$$\pi_t = \frac{1}{\lambda}\Big(\varphi y_t + c_t + (1-\mu)s_t - (1+\varphi)a_t\Big) + \beta\pi^e_{t+1} \tag{40}$$

where a_t is the log permanent relative shock of technology and ς_t is a relative demand shock. e stands for expectation.[18] The relative demand shock can be interpreted as an asymmetric shock to consumption preferences:

$$a_t = \hat{\mathscr{A}}_{At} - \hat{\mathscr{A}}_{Bt} \tag{41}$$

$$\varsigma_t = 2\mu\theta(\hat{\mu}_{At} - \hat{\mu}_{Bt}) \tag{42}$$

[17]Having taken a first-order approximation to arrive at the log-linear equations, these results will only be valid for small disturbances around the steady states, given that some of the levels equations are highly non-linear in nature.

[18]Since the model is log-linear, assuming rational expectations is analytically equivalent to assuming perfect foresight. This is the certainty equivalence result.

and, r is the relative log-real interest rate:

$$r_t = \hat{r}_{At} - \hat{r}_{Bt} = (\hat{i}_{At} - \hat{\pi}^e_{At+1}) - (\hat{i}_{Bt} - \hat{\pi}^e_{Bt+1})$$

We briefly discuss the complete system. Equation (33) defines relative excess return. If z is restricted to be zero, we go back to the standard UIP case.

Equation (34) is the portfolio balance equation. The first term says that excess return is required for consumers to absorb an increase in relative supply of home bonds. An increase in domestic debt f raises excess return as it shifts wealth from home to the foreign country and demand away from domestic bonds. For every unit of wealth, home consumers allocate α in home bonds, but foreign consumers allocate only $(1 - \alpha)$. As a result, f is scaled by $(2\alpha - 1)$. An exchange rate depreciation raises the home value of foreign bonds held by home consumers, which increases the demand of home bonds and drives down the excess return.

The next equation (35) is the aggregate demand. Relative output depends on relative consumption, the real exchange rate and government spending. The consumption term is scaled by $(2\mu - 1)$, accounting for home consumption bias in both countries, and by θ, the share of consumption to GDP. A depreciation of the home exchange rate by a unit increases the producer's price by $(1 - \mu)$ unit relative to the consumption price index, so domestic demand rises by $\mu(1 - \mu)\theta\eta$ for each country, accounting for the consumption proportion spent on home goods μ, the size of consumption θ and elasticity of substitution η. The exchange rate movement affects foreign consumers in the opposite way, so the factor is scaled by 2.

Equation (36) is the Euler equation relating current and future consumption. Current consumption depends negatively on the return on financial wealth. The return on financial wealth is captured by the real interest rate r and excess return z. Excess return is scaled by $(1 - \alpha)$ because domestic consumers need to pay for the excess return on domestic assets held by foreign consumers, which reduces the return on financial wealth to domestic consumers and increases current consumption. An exchange rate appreciation in current period stimulates consumption, but expected appreciation in the future depresses it. The scaling factor $(1 - \mu)$ is, as described above, the increase in PPI relative to CPI.

Equation (37) shows the evolution of domestic debt. The next period's debt is the sum of current debt, interest payment and the government deficit in this period.

Equation (38) shows the evolution of foreign debt, f. The interest payment is the steady state interest f/β plus the excess return of domestic assets held by foreign consumers $(1 - \alpha)z$. Foreign debt rises with an increase in relative consumption. The real exchange rate has two independent effects to the future foreign debt. The direct terms of trade effect $(1 - \mu)\theta$ is positive: when exchange rate depreciates, foreign goods are more expensive. Home consumers will shift their demand away from foreign goods in the way we discussed in (35). These two effects work in opposite directions.

Equation (39) is the simple Taylor rule.

Equation (40) is the forward-looking Phillips curve, where current inflation depends on the marginal cost and discounted expected future inflation. Given the consumption-labour tradeoff, a rise in output indicates an increase in labour supply, keeping productivity constant. This is caused by a rise in the real wage, which increases the marginal cost. Similarly, a rise in consumption also implies a rise in the real wage. The terms of trade matters because the real marginal cost is discounted by PPI but the real wage is discounted by CPI. When terms of trade worsens, nominal wage rises relative to PPI, so real marginal cost increases. Finally, a rise in productivity reduces real marginal cost directly. There is also an indirect reinforcing effect as people desire more leisure and less labour supply, which drives down the real wage and marginal cost.

3.4 Solving the Model

In the complete system, the first six equations are identical to those in the KKV (2008) model. The last two equations, the Taylor rule and the forward-looking Phillips curve respectively, come from the addition of the labour market and nominal rigidities in price-setting. The KKV (2008) model is a real model in which prices are flexible. Here, if we assume that prices are flexible ($\lambda = 0$) and labour supply is vertical ($\varphi = \infty$), then money is neutral and the real variables are orthogonal to monetary forces. We get back to the KKV (2008) model.

KKV (2008) proceed by making two further assumptions. First, they assume that the private sector adjusts to changes in the real interest rate r, which are changed to keep the output fixed at the natural level. Second, they abstract entirely from fiscal policy and assume that any debt is financed by lump-sum taxes so that debt is kept at its steady state level, or $b_t = 0$ at all times, and there are no changes in government expenditure, i.e $g_t = 0$.

We do not make the first assumption of fixed output, for the interest rate is now tied down by the Taylor rule and may not be used to keep output constant. We combine the Phillips curve (40) and the Taylor rule (39) to eliminate the expected inflation at $t + 1$ to give:

$$r_t \;=\; (\gamma - \frac{1}{\beta})\pi_t + \frac{1}{\beta\lambda}\Big(\varphi y_t + c_t + (1-\mu)s_t - (1+\varphi)a_t\Big)$$

A positive relative productivity shock depresses the real marginal cost, reduces relative home inflation and drives down the real interest rate.

Second, we assume that the real government debt remains constant, that is $b_t = 0$ for all t. We follow KKV (2008) and abstract from fiscal policy and assume that $g_t = 0$ for all t. Government spending is financed by lump-sum tax.

After some algebra, the system condenses to the following standard dynamic equation:

$$\begin{pmatrix} f_{t+1} \\ s_{t+1} \\ c_{t+1} \\ \pi_{t+1} \end{pmatrix} = \Theta \begin{pmatrix} f_t \\ s_t \\ c_t \\ \pi_t \end{pmatrix} + \Xi \begin{pmatrix} \varsigma_t \\ a_t \end{pmatrix} \tag{43}$$

where

$$\Theta = \begin{pmatrix} \frac{(3-2\alpha)\alpha-1}{2\alpha\phi} + \frac{1}{\beta} & -\frac{(1-\mu)(2\eta\mu-1)\theta}{B\beta} - \frac{(1-\alpha)^2}{2\alpha\phi} & \frac{2(1-\mu)\theta}{B\beta} & 0 \\ -\frac{2\alpha-1}{2\alpha\phi} & \frac{1-\alpha}{2\alpha\phi} + \frac{2(1-\mu)(2\eta\mu\theta\varphi+1)}{\beta\lambda} + 1 & \frac{2(1+\theta\varphi(2\mu-1))}{\beta\lambda} & 2(\gamma - \frac{1}{\beta}) \\ \frac{(2\alpha-1)(\alpha-\mu)}{2\alpha\phi} & \frac{(1-\alpha)(\mu-\alpha)}{2\alpha\phi} - \frac{(\mu(2\mu-3)+1)(2\eta\mu\theta\varphi+1)}{\beta\lambda} & \frac{(2\mu-1)((2\mu-1)\theta\varphi+1)+\beta\lambda}{\beta\lambda} & \left(\gamma - \frac{1}{\beta}\right)(2\mu-1) \\ 0 & -\frac{(1-\mu)(2\eta\mu\theta\varphi+1)}{\beta\lambda} & -\frac{(2\mu-1)\theta\varphi+1}{\beta\lambda} & \frac{1}{\beta} \end{pmatrix}$$

$$\Xi = \begin{pmatrix} -\frac{1}{B\beta} & 0 \\ \frac{2\varphi}{\beta\lambda} & -\frac{2(\varphi+1)}{\beta\lambda} \\ \frac{(2\mu-1)\varphi}{\beta\lambda} & -\frac{(2\mu-1)(\varphi+1)}{\beta\lambda} \\ -\frac{\varphi}{\beta\lambda} & \frac{\varphi+1}{\beta\lambda} \end{pmatrix}$$

The system can be solved by standard Blanchard-Kahn (1980) technique. In this system, only net foreign debt f is sluggish (state variable); the other three are jump (control) variables. The Blanchard-Kahn condition requires that there is one stable eigenvalue and three unstable. The stable eigenvalue will be linked to the state variable and the unstable to the control. However, as the algebraic expressions for the eigenvalues are intractable, we will examine a calibrated model in the next section. With suitable parameter choice, convergence is ensured and there exists a unique adjustment path.

4 Model Simulation

In this section, we calibrate and simulate our model, compare it with the KKV (2008) model and explain the determinants of the real exchange rate adjustment in response to different shocks. We discuss the role of various parameters in the model and how they affect the simulated results. We also compare the magnitudes of exchange rate movement due to a demand shock and a productivity shock.

In the following simulations, China is the home country and the U.S the foreign. Therefore, we investigate a positive demand shock and a positive productivity shock, which, we believe, are the dominant economic shocks China is now facing.

4.1 Full Model and Equilibrium

Our log-linearised full system is reproduced as follows:

$$z_t = 2r_t + s_t - s^e_{t+1} \tag{44}$$
$$2\alpha\phi z_t = (2\alpha - 1)f_t - (1-\alpha)s_t \tag{45}$$
$$y_t = (2\mu - 1)\theta c_t + 2\mu(1-\mu)\theta\eta s_t + \varsigma_t \tag{46}$$
$$c_t = c^e_{t+1} - r_t + (1-\alpha)z_t + (1-\mu)(s^e_{t+1} - s_t) \tag{47}$$
$$f_{t+1} = (1-\alpha)z_t + \frac{1}{\beta}\left(f_t + 2\frac{\theta}{B}(1-\mu)c_t - \frac{(1-\mu)\theta}{B}(2\mu\eta - 1)s_t - \frac{1}{B}\varsigma_t\right) \tag{48}$$
$$r_t = \gamma\pi_t - \pi^e_{t+1} \tag{49}$$
$$\pi_t = \frac{1}{\lambda}(\varphi y_t + c_t + (1-\mu)s_t - (1+\varphi)a_t) + \beta\pi^e_{t+1} \tag{50}$$

Our model adds two additional nominal equations (49) and (50) to the original KKV (2008) model. The KKV (2008) full system is our first five equations (equations (44) to (48)) with the restriction that $y = 0$.

With these equations, we can find the steady state values of these variables in terms of the shocks ς and a:

$$s = \frac{1}{\Lambda}(-(2\alpha-1)(1+\theta\varphi)\varsigma + 2(2\alpha-1)(1-\mu)(1+\varphi)\theta a) \tag{51}$$
$$f = \frac{1}{\Lambda}(-(1-\alpha)(1+\theta\varphi)\varsigma + 2(1-\alpha)(1-\mu)(1+\varphi)\theta a)] \tag{52}$$
$$c = \frac{1}{\Lambda}\begin{pmatrix} ((2\alpha-1)(1-\mu)(1+\theta\varphi) + B(1-\alpha)(1-\beta)\varphi)\varsigma \\ +[(2\alpha-1)(1-\mu)(2\eta\mu-1)\theta - (1-\alpha)(1-\beta)B](1+\varphi)a \end{pmatrix} \tag{53}$$
$$r = z = \pi = 0 \tag{54}$$
$$y = \frac{1}{\Lambda}\begin{pmatrix} -(1-\alpha)B(1-\beta)\varsigma \\ +\theta(1+\varphi)\begin{bmatrix} (2\alpha-1)(1-\mu)(1+2(\eta-1)\mu)\theta \\ -(1-\alpha)(1-\beta)B(2\mu-1) \end{bmatrix} a \end{pmatrix} \tag{55}$$

where $\{s, f, c, r, z, \pi, y\}$ are the steady state values of $\{s_t, f_t, c_t, r_t, z_t, \pi_t, y_t\}$, and

$$\Lambda = (2\alpha-1)(1-\mu)(2(\eta-1)\mu+1)\theta(1+\theta\varphi) - (1-\alpha)B(1-\beta)(1+(2\mu-1)\theta\varphi).$$

For β close to unity, Λ is positive.

4.2 Calibration

We need to calibrate ten variables. They are: home consumption bias μ; home asset bias α; the discount factor β; degree of international capital mobility ϕ; elasticity of substitution between home and foreign goods η; share of consumption in the economy θ; ratio of the stock of government bonds to GDP B; the Taylor rule coefficient on inflation γ; price stickiness parameter λ and inverse of labour supply elasticity φ.

Parameter	μ	θ	α	B	β	γ	ϕ	λ	η	φ
Value	0.85	0.75	0.8	3	0.99	1.5	1	2	1.2	1

Table 1: Parameter Values for Simulation

Table 1 summarises the choice of parameter values in our calibration. Most of these values are determined by values either often used in the literature or using prior empirical evidence. We explain the parameter choices below.

- Home consumption bias μ: We consider empirical estimates that use a gravity model of trade to measure home bias. Using U.S. and Canada data for 1993-1996, Helliwell (1998) finds that the unexplained home bias is about 12 times. Wei (1998) and Evans (1998) test home bias for other OECD country pairs. Wei finds that the average bias may be as low as 2.5 times, but the estimate could be biased downward due to his exclusion of the service sector. Evans find values between Wei's and Helliwell's. We use $\mu = 0.85$ in our calibration, corresponding to a bias of 5-6 times.

- Home asset bias α: Blanchard, Giavazzi and Sa (2005) estimate that $\alpha = 0.77$ for the U.S and 0.7 for the rest of the world, where they use a ratio of financial assets to GDP of about 2 for the non-U.S world. For China, the GDP in China is about \$5.2 trillion in 2008.[19] A reasonable estimate for Chinese financial wealth would be \$10.4 trillion. As for the Chinese holding of U.S assets, a recent estimate by Setser (2009) suggests a value of around \$1.5 trillion. Given the \$2 trillion FX reserve in China and about a 60-70% share in U.S assets, the value seems reasonable. Then α would be $1 - (1.5/10.4) = 85\%$. We use $\alpha = 0.8$.

- Discount factor β: We assume that the quarterly discount factor is $\beta = 0.99$, implying a steady state asset return of 4.1% per annum.

- International capital mobility ϕ: We assume $\phi = 1$. It implies an increase of the expected excess return of 1% increases the desired share of home assets by 0.8% (More formally, we differentiate equation (24), so $d\alpha_{At} = \alpha\phi Z_{At}^{\phi-1} dZ_{At}$, and then substitute $\alpha = 0.8$ and the steady state value $Z = 1$.). We also simulate with $\phi = 10$ (10 times more sensitive) and to study the effect of capital mobility to the system. The choices roughly correspond to the medium and high mobility in Blanchard, Giavazzi and Sa (2005). KKV (2008) also use similar parameterisation.

- Elasticity of substitution between home and foreign goods η: There is not much consensus about the appropriate value for η. Backus, Kehoe, and Kydland (1994) and Engel and Matsumoto (2009) use $\eta = 1.5$. In some real business cycle models, η is even lower (see Heathcote and Perri (2002), for example). KKV (2008) use $\eta = 1$, the Cobb-Douglas preference. However, studies on price markups give a larger estimate of η (Obstfeld and Rogoff 2000). We use $\eta = 1.2$ in our calibration.

[19]Source: World Bank China Quarterly Update (March 2009).

- Share of consumption in the economy θ: A plausible parameterisation would be $\theta = 0.75$, so the government's share is 0.25.

- Ratio of the stock of bonds to GDP B: Using data of total debt in the U.S and the rest of the world, Blanchard, Giavazzi and Sa (2005) estimate a number of around 3.5. KKV (2008) set this to 3.[20] We use $B = 3$ in our calibration as we believe that the Chinese bond market is not as well-developed.

- Taylor rule coefficient on inflation γ: The Taylor principle suggests that nominal interest rate should move by more than inflation, that is, γ should be greater than unity. We assume $\gamma = 1.5$, as in Gali and Monacelli (2002).

- Price stickiness λ: in the literature, most calibrations with Calvo pricing assume that the nominal price lasts for four quarters (Engel and Matsumoto 2005). This implies roughly $(1 - \kappa) = 1/4$, or one fourth of the firms resets prices every quarter and $\lambda = \kappa/[(1 - \kappa)(1 - \beta\kappa)] \approx 12$. However, evidence produced by Bils and Klenow (2004) suggests that the average life of a price is closer to two quarters, which implies $\lambda = 2$. We use $\lambda = 2$ in our calibration.[21]

- Inverse of labour supply elasticity φ: We assume quadratic labour disutility, which implies $\varphi = 1$.

4.3 Demand shock

We first look at a once and for all positive demand shock, a unit increase of ς, and analyse a shift of demand from U.S to Chinese goods. The shock may be related to changing consumer preferences. We seek to understand how the bilateral exchange rate and foreign debt position are affected with the simulations below.

Figure 1 shows the adjustment in the KKV (2008) model.[22] In response to a positive demand shock in China, prior to any adjustment in the stock of net debt, the renminbi real exchange rate appreciates. To Chinese consumers, the renminbi value of U.S assets falls. As a result, due to home asset bias, there will be an excess demand for U.S assets which drives down the excess return for U.S assets. On the other hand, Chinese consumers will sell Chinese assets. This drives up the excess return for Chinese assets. For U.S consumers, since the U.S exchange rate is depreciated, they desire to hold more U.S assets, also driving down the excess return for U.S assets.

[20] In fact, the definition of B in the model is associated with public debt, because there is no capital accumulation and no private debt. However, the analysis of capital movement responding to expected excess return applies to private debt too. We use total debt figures in the calibration, which explain the large value of B.

[21] In fact, as the Phillips curve has no persistence, the choice of λ has only a tangential effect on the simulations. This is a drawback of the present model.

[22] KKV (2008) focus on the U.S current account deficit. They simulate a negative demand shock of magnitude unity. Our simulation result is a mirror image of their simulation.

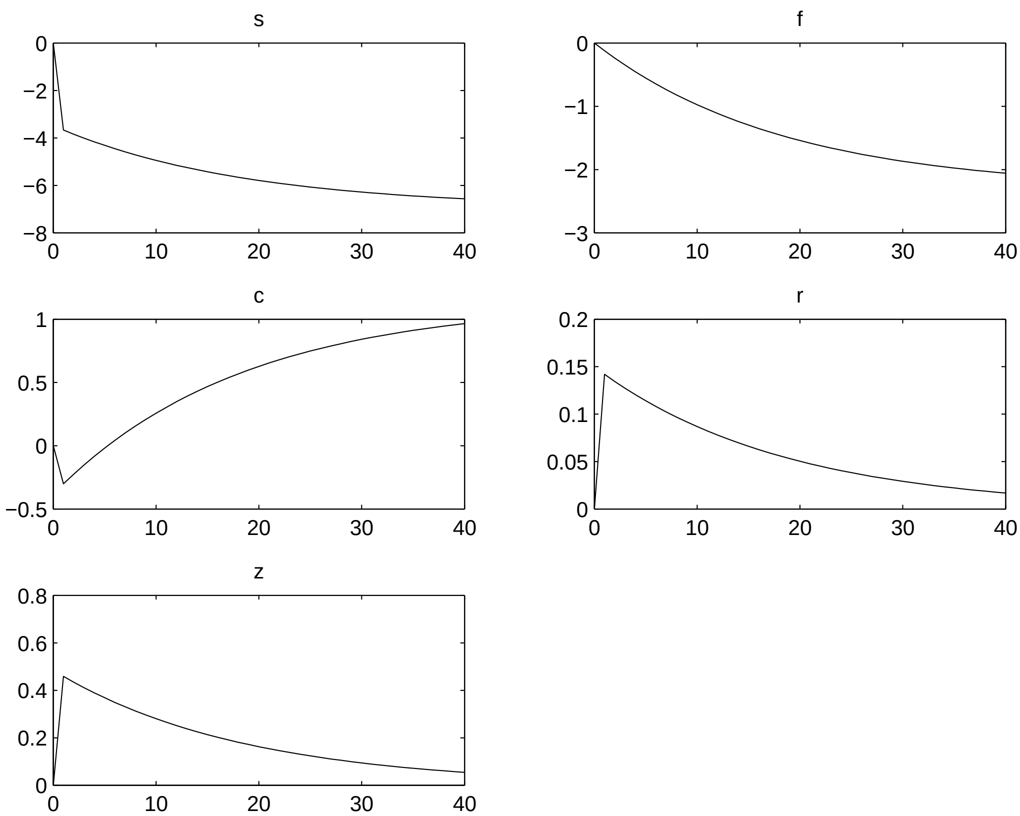

Figure 1: Simulation of the KKV (2008) model with a unit positive demand shock

Owing to imperfect capital mobility, the Chinese renminbi does not appreciate initially to its equilibrium. Along the adjustment path, the real exchange rate is undervalued, so net foreign assets in China gradually builds up. In other words, net debt f is falling. The implication is the Chinese reserve will continue to build up so long as the currency is still undervalued. This currency undervaluation and the associated gradual build-up of foreign exchange reserve is a central feature of the stock-flow adjustment of the model.

On the other hand, the exchange rate appreciation leads to a rise in consumption in China. KKV (2008) assume that the real interest rate adjusts to keep output constant, so according to the Euler equation the real interest rate will have to rise initially to keep consumption low. The rise in the Chinese real interest rate raises excess return further and aggravates the initial appreciation of the Chinese renminbi.

In the new equilibrium, excess return goes back to zero, and interest rates between the two countries equalise. The Chinese renminbi exchange rate will be permanently higher and foreign debt lower. Therefore, along the adjustment path, we see an initial jump in the real exchange rate, followed by a further continuous appreciation. The dynamics comes from the relaxation of the UIP in the financial market.

Figure 2 shows the simulation of the present model with nominal rigidities. To begin, we assume that economic agents do not discount the future ($\beta = 1$) and steady state world interest rate is zero for a reason which will soon be evident. This assumption will be relaxed later.

Comparing the results from Figure 1 and Figure 2, the real interest rate adjustment in this case is smaller. This is intuitive: KKV (2008) assume that real interest rate adjust to keep output unchanged. However, in our model, output is not always in equilibrium: the real interest rate is determined by the Taylor rule. In response to a positive demand shock, the renminbi appreciates. The appreciation itself has two *deflationary* effects. First, terms of trade improves so exports is less competitive. Second, on the asset side, an appreciation of the renminbi decreases the value of U.S assets held by Chinese consumers in Chinese renminbi terms. With these offseting effects, the real interest rate set by the monetary policy rule rises by less than what would be in the KKV (2008) case. Finally, the new steady state foreign debt position is smaller than in the original KKV (2008) model and steady state consumption less positive.

Figure 3 looks at the simulation in which consumers and firms discount future utility. The long-run real exchange rate and foreign debt positions are dependent on the value of β, so the steady state levels are different from that of Figure 2. It is surprising that the steady state output is below the original equilibrium, despite a higher domestic consumption. This result is robust to alternative parameter choices, observing that in the equilibrium equations (51) and (55), the ς coefficient of s and y must have the same sign. How do we account for this seemingly unintuitive result?

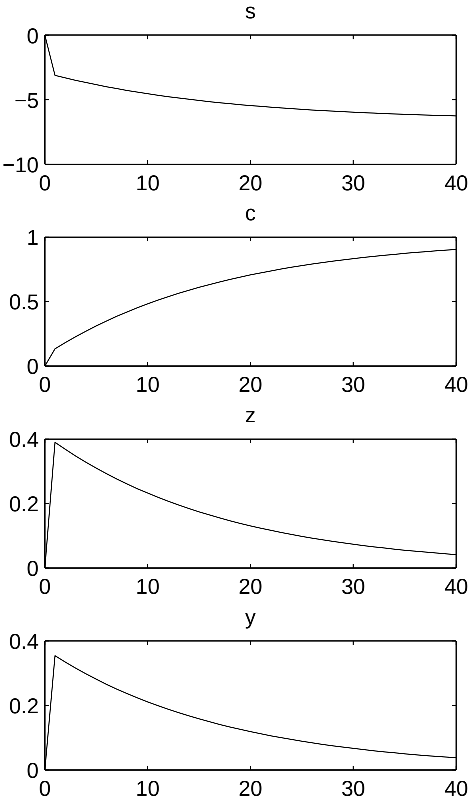

Figure 2: Simulation of our model with a unit positive demand shock, with the restriction $\beta = 1$

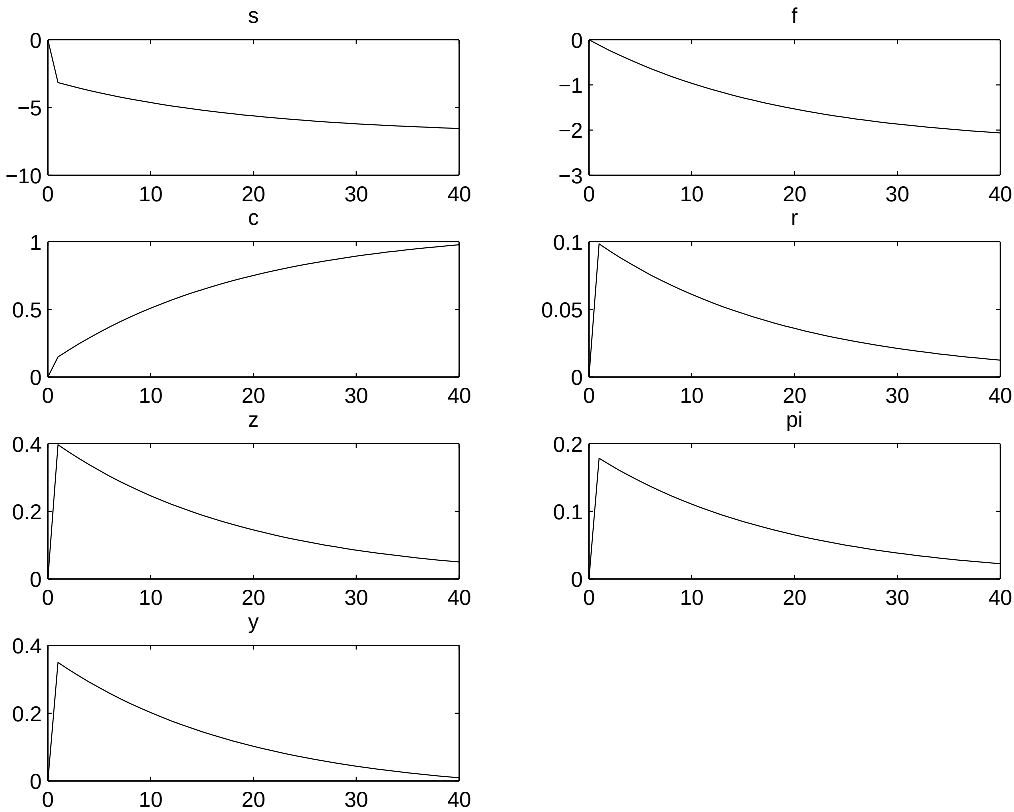

Figure 3: Simulation of our model with a unit positive demand shock

We answer this question in two steps. First, recall the intratemporal optimal condition and the linear production function:

$$\varphi l_t + c_t = (w_t - p_t) \tag{56}$$

$$y_t = a_t + l_t \tag{57}$$

We know from equation (55) and Figure 2 that when $\beta = 1$, a positive demand shock does not change the long-run output, but it raises relative consumption. As no wage rigidities is assumed, equation (56) implies that the real wage rises as much as consumption does, which keeps labour supply and output unchanged. Both consumption and the real wage are affected in the same way by the real exchange rate appreciation and interest rate rises.

Then, why does a change of β matter? The influence of β to the position of the steady state only comes from equation (48). The steady state version is given below:

$$f = \frac{1}{\beta}\left(f + 2\frac{\theta}{B}(1-\mu)c - \frac{(1-\mu)\theta}{B}(2\mu\eta - 1)s - \frac{1}{B}\varsigma\right) \tag{58}$$

The β here has the interpretation of the inverse of one plus the steady state real interest rate across U.S and China. When β is less than one, the steady state real interest rate is positive.[23] Along the saddle path, Chinese debt is decreasing, so Chinese consumers earn a positive interest payment from the rise in national claim against the U.S consumers. As a result, consumption in China rises more than the real wage. According to (56), consumers in China will choose more leisure and less work, and output must fall. This also explains why consumption now increases more than unity.

In Figure 4, we analyse the role of the responsiveness of capital flows, ϕ, to excess return. We plot the impulse response of a unit demand shock with ϕ equals 1, 10, 100 respectively. A higher ϕ implies more capital mobility, and when ϕ equals 100, the model behaves like the Dornbusch (1976) model which assumes the uncovered interest parity. When capital is more mobile, the initial jump in exchange rate is bigger. The reason is the following: when assets are more substitutable, the excess return will be smaller, so consumers expect smaller real exchange rate changes along the adjustment path. Therefore, a larger initial exchange rate appreciation is necessary to put the system in the non-explosive equilibrium path. The initial appreciation alone has a deflationary impact through the goods market and capital market channels which partly offsets the inflation caused by the rise in demand. As a result, the real interest rate does not have to rise as much as when capital is less mobile, and consumption exhibits a larger jump. Subsequently, the speed of adjustment is slower, since the real interest rate and excess return are smaller. The important implication is that ϕ governs the speed of renminbi appreciation.

[23] When β is too small, or the real interest rate too large, the system is unstable, as the country assets or debt will explode.

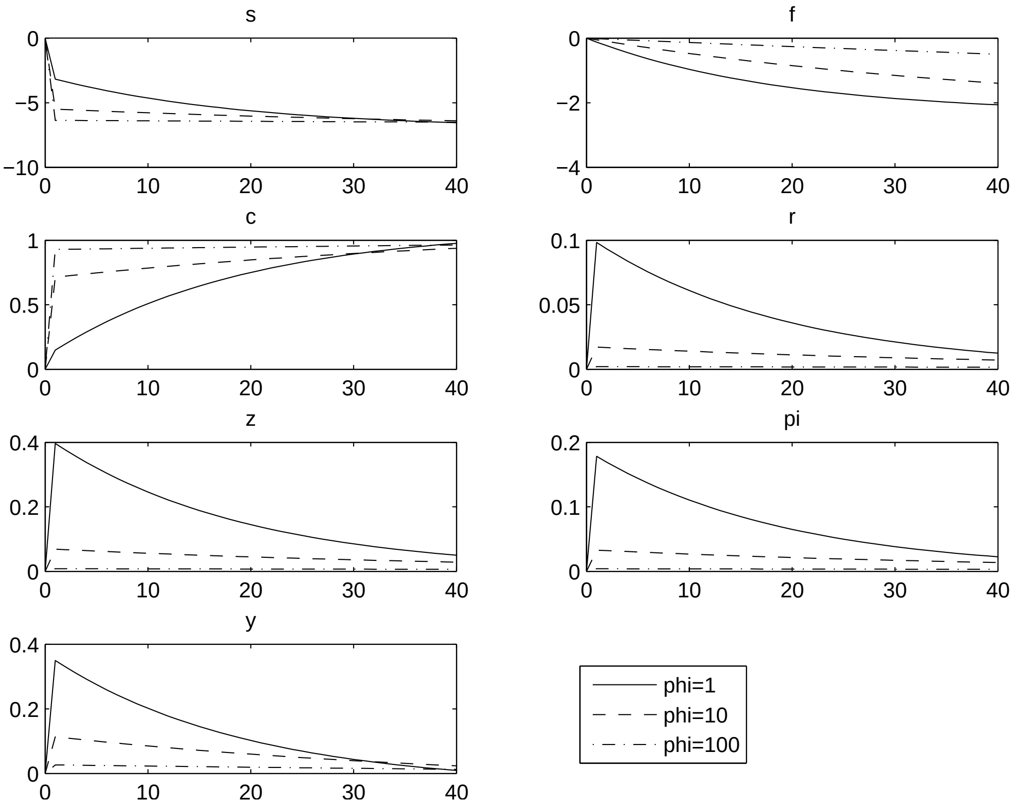

Figure 4: Simulation of our model with a unit positive demand shock, $\phi = 1, 10, 100$

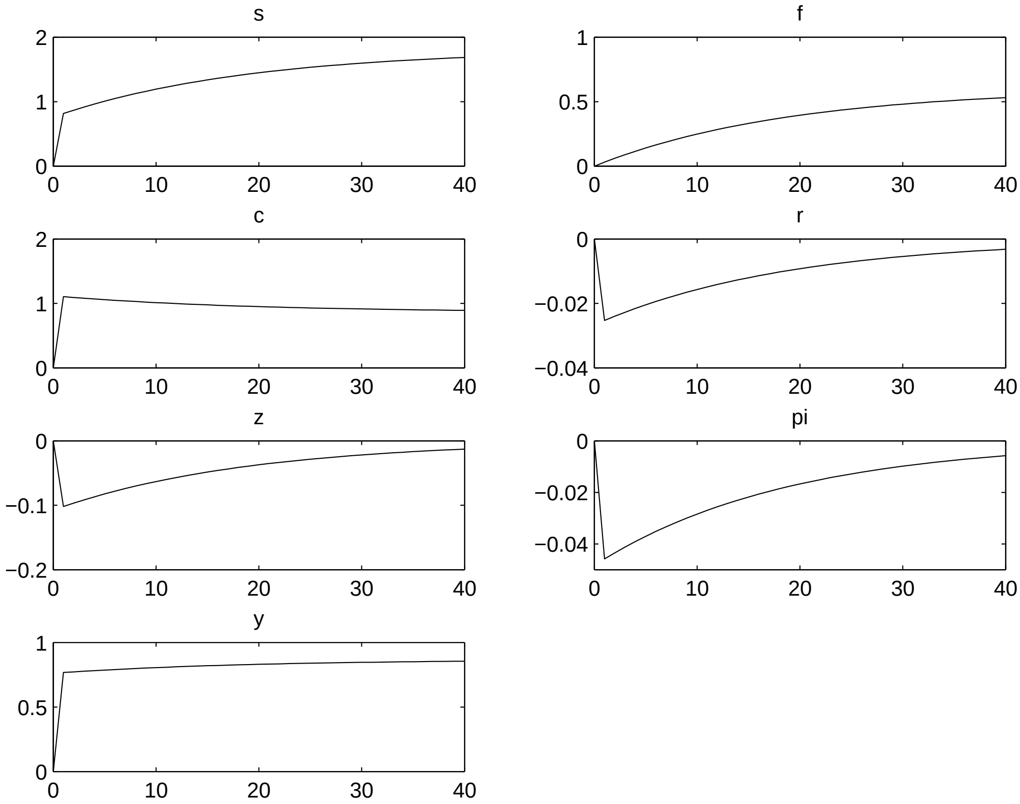

Figure 5: Simulation of our model with a unit positive productivity shock

Varying ϕ does not alter the steady state values. The steady state equations (51) to (55) do not depend on ϕ. This is because the deviation from the UIP is temporary. In the steady state, the return on assets from the two countries must equalise. ϕ determines the magnitude of the initial jump and the subsequent speed of adjustment. Thus, the capital mobility parameter ϕ can be viewed as a crude instrument to model the degree of capital controls in China. Notice that the impulse responses correspond to the two-step exchange rate adjustment proposal by Goldstein and Lardy (2003, 2008). By allowing some initial jump in the real exchange rate, subsequent deviation away from the U.S interest rate can be reduced, and volitility in consumption will be dinimished.

4.4 Productivity Shock

In this section, we analyse the effect of a once and for all productivity shock, or a unit increase in a. Given the fact that China is a developing country with large FDI inflow, we would reasonably expect some technology transfer taking place during the process and productivity in China grows in relative terms.

Figure 5 shows the response of the system to a positive relative productivity shock in China. The shock has two consequences. First, the Chinese yuan *depreciates*. The depreciation enables China to sell the increased production to the rest of the world. Second, a rise in productivity reduces the marginal cost of production, which leads to a deflation in the Chinese economy. When the Chinese yuan depreciates, there will be an excess demand of Chinese assets through denomination effect and the return of Chinese assets has to fall. The real interest rate has to fall initially below the equilibrium level, leading to an overshoot in consumption.

In the long run, both consumption and output in China rise, but by less than unity. Consumption rises by less than one-to-one because the terms of trade worsens, which makes foreign goods more costly. The latter outcome depends on the labour supply and demand. The demand for labour depends on the PPI, while the supply of labour depends on the CPI. A positive productivity shock increases demand of labour and real wages. But the labour supply decreases due to an income effect. In our simulation, the labour spply curve is steep, so the total effect is a fall in labour. Thus, the rise in output is less than one.

We believe that China is being hit by both demand shocks and productivity shocks. It is important, therefore, to notice that a positive producitivty shock in China, *ceteris paribus*, offsets partly the effects of a shift of demand from the U.S to China and reduces the need of yuan exchange rate appreciation.

However, we observe, by comparing Figure 3 and Figure 5, that the long-run terms of trade adjustment is much larger in the case of a demand shock. Figure 6 puts both shocks of same magnitude together.

If China is hit by the two shocks of the same magnitude simultaneously, the Chinese renminbi exchange rate will jump downwards initially, ensued by further appreciation to the new steady state. This is because the effect of the demand shock dominates. The initial appreciation leads to a fall in demand for Chinese assets. The return of Chinese assets rises. The demand shock causes overheating in the Chinese economy, and the monetary authority has to raise interest rate to cool down the economy. Consumption jumps up initially due to an increase in both domestic productivity and demand for domestic goods. Currency appreciation makes foreign goods cheaper, which stimulates consumption in China. Consumption rises further along the saddle path as interest rate is falling and the assets are reallocated from the U.S to China. Output rises by less than unity, again due to the consumption-labour tradeoff.

We do not claim that we know the relative sizes of the demand shocks and productivity shocks in China. Nevertheless, we can compute the shock in demand required to offset the real exchange rate deprecation caused by the productivity shock. From the steady state equation (51), we set the real exchange rate s to zero and get:

$$\varsigma = \frac{2(1-\mu)(1+\varphi)\theta}{1+\theta\varphi} a \tag{59}$$

The fraction in (59) is smaller than one because $\mu \geq 1/2$ and $\theta \leq 1$. It reveals two

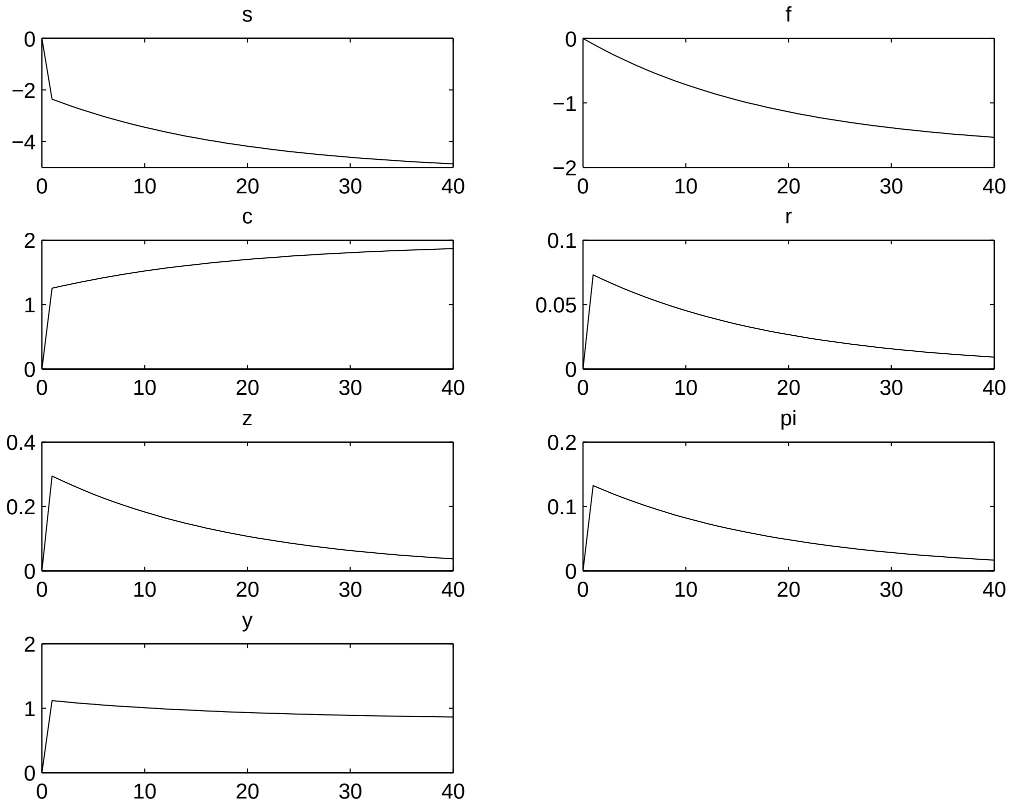

Figure 6: Simulation of our model with a unit positive demand shock and a unit positive productivity shock

independent reasons why the demand shock has a larger effect on the real exchange rate relative to the productivity shock. Firstly, a demand shock is entirely about consumption shifting from one country to another, but a productivity shock affects a country as a whole, with larger than half of the effect absorbed domestically because of home consumption bias μ. Secondly, we assume that the governments only purchase goods from their own country, so part of the shock in productivity is absorbed by the government. Therefore, the demand shock dominates the real exchange rate adjustment in Figure 6.

Substituting our parameters in Table 1 into equation (59), we show that a positive demand shock with a magnitude of around 26% (in logarithmic term) of that of the productivity shock would be sufficient to offset the depreciation in exchange rate. Then, using equation (42),

$$\varsigma = 2\mu\theta(\hat{\mu}_{At} - \hat{\mu}_{Bt}) \tag{60}$$

we can back out the shock in home consumption bias μ required to offset the exchange rate depreciation caused by a productivity shock. If we assume that the shift in home consumption bias in both countries are equal in magnitude but in the same direction towards Chinese goods, i.e. $\hat{\mu}_{At} = -\hat{\mu}_{Bt}$, then a 10% shift in home consumption bias in each country would offset the depreciation. In other words, a shift in consumption preference in both countries by 10% offsets the depreciation induced by a unit increase in productivity.[24]

What are the sizes of these relative shocks in reality? van Ark et al. (2008) estimate that the TFP growth in the U.S in 1995-2005 is about 2.3% annually. Estimations with alternative assumptions give quite consistent results ranging between 1.5% to 2.5%. As for the TFP growth in China, estimates in the literature are rather dispersed, ranging from 0 to 9% (Hu and Zheng 2006, Heshmati and Shiu 2006, He and Kuijs 2007). The wide range of estimates may be related to the provincial disparity and sectoral divergence in growth. The quality of data also affects the results. For instance, one way to measure TFP growth is to use growth accounting methods. However, Blanchard and Giavazzi (2005) point out that there is considerable uncertainty about capital accumulation using a perpetual inventory method. We use a number of 3.5% for the TFP growth of China, averaging the findings from the papers above.

We would like to establish a connection between the relative productivity shocks and demand shocks. Doing so properly would require proper modeling. We do not attempt it here. Krugman (1979) indicates a way to think about the issue. He develops an international trade model between a developed North and a developing South. The North produces only new goods and the South can only produce old. New goods become old with a time lag when technology is transferred to the South. As South acquires the technology from the North to produce one more good, the production of that good will be reallocated to the South, and demand also shifts from North to South. The North will

[24] Alternatively, if we assume that government purchase resembles private consumption with home bias μ, a demand shock of magnitude 0.3 or a 9% shift in home consumption bias in each country can offset the depreciation caused by a unit increase in productivity.

keep innovating to gain monopoly rent, and this increases the diversity of goods.

Krugman's (1979) setup is not quite consistent to ours in at least three ways. First, his technological change refers to an increase in number of goods whereas ours is a rise in productivity for all production. Second, in his world, there are finite goods, but our world has an infinite spectrum of goods. Third, our model is symmetrical but Krugman's is highly asymmetric. Nevertheless, we still hope to parameterise the variables to our setup and see what our model implies.

Using the TFP growth data discussed above, we assume that the innovation rate in the U.S is 2% per year. Out of the 3.5% productivity growth in China, we do not know exactly how much can be attributed to innovation and how much to learning from other countries. Very loosely, if we are willing to assume, for instance, 1.5% is attributed to learning, then the shift in relative demand is $(1.5\% - (-1.5\%)) = 3\%$ per year from the U.S to China, and change in relative productivity is $(3.5\% - 2\%) = 1.5\%$. Putting these values into our model, we get an appreciation of the Chinese real exchange rate by about 18% in the steady state, while the initial jump is about 8%.

We recognise that these estimates are not as robust to alternative assumptions as we would like them to be. For example, if the productivity growth in China is split into 1% learning and 2.5% innovation, then the initial and total real exchange rate appreciation will be 5% and 11% respectively. There are large uncertainties about the real exchange rate appreciation in China, just as other estimation approaches which we reviewed in section 2 suggest.

5 Conclusion

This paper explored a theoretical approach to understand the appreciation of the Chinese yuan, focusing on imperfect substitutability between assets. To capture imperfect substitutability between assets, our model relaxes the UIP and allows return on different assets to differ in the short run. This gives the initial jump and gradual appreciation dynamics of the real exchange rate movement in response to demand shocks and productivity shocks, which help us think about the Goldstein and Lardy (2003, 2008) two step appreciation procedures. The extent of initial jump and the subsequent rate of appreciation depend on the degree of capital mobility. This is consistent with the notion that the Chinese authorities should reform the exchange rate regime in tandem with other policies, such as gradual capital account liberalisation. Another important implication of this model, which is much less frequently mentioned in the policy circle, is that positive productivity shocks in China reduce the extent of yuan exchange rate appreciation required to rebalance the current accounts in China and in the U.S.

While we have mentioned some limitations of our model previously, this section discusses some unresolved problems and provides some suggestions for further research. One major limitation is we abstract from government spending and debt. We argue in section

2 that fiscal expansion is conducive to the exchange rate transition and internal rebalance. The present analysis fails to capture this channel. A minor change could be made so that the government follows a feedback rule of this kind:

$$g_t = \psi_\pi \pi_t + \psi_s s_t - \psi_b b_t \tag{61}$$

in which an appropriate choice of $\{\psi_\pi,\ \psi_s,\ \psi_b\}$ ensures debt attainability and convergence. A more rigorous treatment might involve microfounding the fiscal rule.

Besides, we do not have non-tradable goods in our model. For this reason, we assume the law of one price and the Balassa-Samuelson effect is not brought up in the model. In a fast-growing economy like China with high productivity growth, we believe this effect on the real exchange rate and the real wage is not negligible. Blanchard and Giavazzi (2005) models China with tradable, non-tradable and agricultural goods in light of the fact that some good prices are subject to price controls. This is another interesting research direction.

We choose the forward-looking New-Keynesian Phillips curve in the model mainly because of its analytical tractability. However, it is well-known that the Phillips curve has no persistence and does not fit empirical data well. Another drawback is that the simulations in section 4 are not sensitive to λ, the nominal rigidity, again for the reason that inflation is a jump variable. A backward-looking rule-of-thumb pricing rule may therefore fit empirical data better as well as improve the simulation, but the question is how to justify 'indexing' or 'rule-of-thumb' price-setting behaviours. Steinsson (2003) provides the details of the Phillips curve.

Furthermore, the present analysis is based on a two-country system between China and the U.S. The symmetric equilibrium assumes that the two economies are identical in every respect. This is far from the reality. For instance, current U.S GDP is about \$13 trillion, roughly 2.5 times as large as Chinese GDP.[25] Since the Chinese yuan is undervalued also in a trade-weighed basis and the current basket peg is moving towards a wider range of currencies (Frankel, 2009), an alternative modeling choice may be modelling a small open economy as in Gali and Monacelli (2005). Another consideration is about moving away from the symmetric equilibrium. For instance, Devereux and Genberg (2007) allow for different pricing policies and intermediate goods imports in China in their two-country framework. Future research may consider some other differences between countries.

This work is part of what is necessary to the full understanding of the global imbalance that we are facing. As mentioned in the beginning, what is left behind is a more fundamental issue, namely the global saving glut. Consider the textbook Swan diagram model. Suppose, for some reasons, Chinese households desire to save a lot. The resulting saving and investment gap drives down the interest rate, which leads to a depreciation of the Chinese currency. The Chinese authority does this by keeping a low nominal exchange rate. Adam and Vines (2009) discuss the imbalance between China and the U.S

[25] Source: National Economic Accounts, Bureau of Economic Analysis (2008).

in relation to the exchange rate and saving rate assuming China sets the exchange rate as a Stackelberg leader and the U.S sets the world interest rate to maintain full employment. In our model, however, there is no discussion regarding the high savings in China relative to that in the U.S. In fact, given our assumption of forward-looking consumers and Ricardian equilivalence, not much can be said about household savings. Furthermore, our model abstracts entirely from the accumulation of physical capital, so we are unable to discuss excessive firm savings in China, which is the main driving factor behind the high savings rate. There are several approaches to enable the model to analyse the savings and exchange rate issue properly. First, we may modify the setup to model physical capital accumulation in the firms, and will assume that the Chinese firms have low dividend payouts relative to the U.S firms. Second, an overlapping generations setup may be introduced to investigate the intergenerational saving behaviour of the households. In either case, we will have to abandon the symmetry between the two countries to explore the implications of the imbalance. But the bottom line is that it is necessary to build a model that spells out the relationship between saving and exchange rate in order to have a more informed analysis of the global imbalance. We leave this for our future research.

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A Appendix

A.1 Deriving Calvo Pricing Phillips Curve

The derivation follows closely to Walsh (2003). Let $\mathscr{Q}_{At} = \tilde{P}_{AHt}/P_{AHt}$, $\hat{\pi}_{At} = \ln(P_{AHt}/P_{AHt-1})$. From (16),

$$\ln \mathscr{Q}_{At} = \hat{\mathscr{Q}}_{At} = \frac{\kappa}{1-\kappa}\hat{\pi}_{At} \tag{62}$$

and from (14),

$$\mathscr{Q}_{At} = \frac{\tilde{P}_{AHt}}{P_{AHt}} = \frac{\epsilon}{\epsilon-1}\frac{\mathscr{E}_t \sum_{i=0}^{\infty}(\beta\kappa)^i \left(\frac{P_{AHt+i}}{P_{AHt}}\right)^{\epsilon} \frac{Y_{At+i}}{C_{At+i}} MC_{At+i}}{\mathscr{E}_t \sum_{i=0}^{\infty}(\beta\kappa)^i \left(\frac{P_{AHt+i}}{P_{AHt}}\right)^{\epsilon-1} \frac{Y_{At+i}}{C_{At+i}}}$$

$$\mathscr{Q}_{At}\left(\mathscr{E}_t \sum_{i=0}^{\infty}(\beta\kappa)^i \left(\frac{P_{AHt+i}}{P_{AHt}}\right)^{\epsilon-1} \frac{Y_{At+i}}{C_{At+i}}\right) = \frac{\epsilon}{\epsilon-1}\left(\mathscr{E}_t \sum_{i=0}^{\infty}(\beta\kappa)^i \left(\frac{P_{AHt+i}}{P_{AHt}}\right)^{\epsilon} \frac{Y_{At+i}}{C_{At+i}} MC_{At+i}\right)$$

The steady state value $\mathscr{Q} = 1$, therefore $MC_A = (\epsilon-1)/\epsilon$. The left hand side log-linearisation around the steady state is:

$$\frac{Y_A}{C_A}\mathscr{E}_t \sum_{i=0}^{\infty}(\beta\kappa)^i \left((\epsilon-1)(\hat{P}_{AHt+i} - \hat{P}_{AHt}) + \hat{Y}_{At+i} - \hat{C}_{At+i}\right) + \hat{\mathscr{Q}}_{At}\frac{Y_A}{C_A}\frac{1}{1-\beta\kappa} \tag{63}$$

and the right hand side is approximated by:

$$\frac{Y_A}{C_A}\mathscr{E}_t \sum_{i=0}^{\infty}(\beta\kappa)^i \left(\epsilon(\hat{P}_{AHt+i} - \hat{P}_{AHt}) + \hat{Y}_{At+i} - \hat{C}_{At+i} + \hat{MC}_{At+i}\right) \tag{64}$$

Eliminating terms:

$$\begin{aligned}
\frac{\hat{\mathscr{Q}}_{At}}{1-\beta\kappa} &= \mathscr{E}_t \sum_{i=0}^{\infty}(\beta\kappa)^i \left((\hat{P}_{AHt+i} - \hat{P}_{AHt}) + \hat{MC}_{At+i}\right) \\
\frac{\hat{\mathscr{Q}}_{At}}{1-\beta\kappa} &= \mathscr{E}_t \sum_{i=0}^{\infty}(\beta\kappa)^i \left(\hat{P}_{AHt+i} + \hat{MC}_{At+i}\right) - \frac{\hat{P}_{AHt}}{1-\beta\kappa} \\
\hat{P}_{AHt} + \hat{\mathscr{Q}}_{At} &= (1-\beta\kappa)\mathscr{E}_t \sum_{i=0}^{\infty}(\beta\kappa)^i (\hat{P}_{AHt+i} + \hat{MC}_{At+i}) \\
\hat{P}_{AHt} + \hat{\mathscr{Q}}_{At} &= (1-\beta\kappa)(\hat{P}_{AHt} + \hat{MC}_{At}) + \beta\kappa(\mathscr{E}_t\hat{P}_{AHt+1} + \mathscr{E}_t\hat{\mathscr{Q}}_{At+1}) \\
\hat{\mathscr{Q}}_{At} &= (1-\beta\kappa)\hat{MC}_{At} + \beta\kappa(\mathscr{E}_t\hat{\pi}_{At+1} + \mathscr{E}_t\hat{\mathscr{Q}}_{At+1})
\end{aligned}$$

Making use of (62)

$$\begin{aligned}
\frac{\kappa}{1-\kappa}\hat{\pi}_{At} &= (1-\beta\kappa)\hat{MC}_{At} + \beta\kappa(\mathscr{E}_t\hat{\pi}_{At+1} + \frac{\kappa}{1-\kappa}\mathscr{E}_t\hat{\pi}_{At+1}) \\
\hat{\pi}_{At} &= \frac{(1-\kappa)(1-\kappa\beta)}{\kappa}\hat{MC}_{At} + \beta\mathscr{E}_t\hat{\pi}_{At+1} \\
\hat{\pi}_{At} &= \frac{1}{\lambda}\hat{MC}_{At} + \beta\mathscr{E}_t\hat{\pi}_{At+1}
\end{aligned}$$

where λ is defined as $\kappa/[(1-\kappa)(1-\kappa\beta)]$. From equations (8), (15) and the production function (12)

$$\begin{aligned} MC_{At} &= \frac{(1-\tau)w_{At}}{P_{AHt}\mathscr{A}_{At}} = \frac{\nu L_{At}^{\varphi} C_{At} P_{At}}{P_{AHt}\mathscr{A}_{At}} && (65) \\ \therefore \hat{MC}_{At} &= \varphi\hat{L}_{At} + \hat{C}_{At} + \hat{P}_{At} - \hat{P}_{AHt} - \hat{\mathscr{A}}_{At} \\ &= \varphi(\hat{Y}_{At} - \hat{\mathscr{A}}_{At}) + \hat{C}_{At} + \hat{P}_{At} - \hat{P}_{AHt} - \hat{\mathscr{A}}_{At} \\ &= \varphi\hat{Y}_{At} + \hat{C}_{At} + \hat{P}_{At} - \hat{P}_{AHt} - (1+\varphi)\hat{\mathscr{A}}_{At} && (66) \end{aligned}$$

where a_t is the permanent relative shock of technology. Therefore, the linearised Phillips curve has the following form:

$$\hat{\pi}_{At} = \frac{1}{\lambda}\Big(\varphi\hat{Y}_{At} + \hat{C}_{At} + \hat{P}_{At} - \hat{P}_{AHt} - (1+\varphi)\hat{\mathscr{A}}_{At}\Big) + \beta\mathscr{E}_t\hat{\pi}_{At+1} \quad (67)$$

A.2 Log-linearising the section 3 model

In this appendix, we derive in detail the log-linearised model. First from the aggregate price equation (3) and the definition of the real exchange rate (10), we get:

$$\begin{aligned} \hat{P}_{At} &= \mu\hat{P}_{AHt} + (1-\mu)\hat{P}_{AFt} \\ &= \hat{P}_{AHt} + (1-\mu)(\hat{P}_{AFt} - \hat{P}_{AHt}) \\ &= \hat{P}_{AHt} + (1-\mu)\hat{S}_{At} && (68) \end{aligned}$$

Using a similar procedure, we can write the log-deviation of foreign consumption price in terms of the log-deviation of foreign production prices and the real exchange rate:

$$\hat{P}_{Bt} = \hat{P}_{BFt} - \mu\hat{S}_{Bt} = \hat{P}_{BFt} + \mu\hat{S}_{At} \quad (69)$$

Next, we define the real interest rate r_{Jt} as follows

$$1 + i_{Jt} = (1 + r_{Jt})\frac{P^e_{JHt+1}}{P_{JHt}} \quad (70)$$

We write the log real interest rate as the difference between nominal interest rate and producer's price inflation.

$$\hat{r}_{Jt} = \hat{i}_{Jt} - \hat{\pi}^e_{Jt+1} \quad (71)$$

where we define $\hat{x}_{Jt} = (x_{Jt} - x_J)/(1 + x_J)$ for $x = \{i, r, \rho\}$ be the deviation from steady state interest rates. It is easy to see that, in the steady state $r_J = i_J = \rho_J = 1/\beta$. The home return on wealth equation (22) is log-linearised as follow:

$$\begin{aligned} (1+\rho_A)\hat{\rho}_{At} &= (1+i_A)\hat{i}_{At} + (1+i_A)(\alpha\hat{\alpha}_{At} + (1-\alpha\hat{\alpha}_{At}) - (1-\alpha)\hat{Z}_{At}) \\ \hat{\rho}_{At} &= 1 + \hat{i}_{At} - (1-\alpha)\hat{Z}_{At} && (72) \end{aligned}$$

Log-linearisation of the excess return (19):

$$\begin{aligned}
\hat{Z}_{At} &= \hat{i}_{At} - \hat{i}_{Bt} + (\hat{P}^e_{BFt+1} - \hat{P}^e_{AHt+1}) - (\hat{P}_{BFt} - \hat{P}_{AHt}) \\
&= (\hat{r}_{At} - \hat{P}_{AHt} + \hat{P}^e_{AHt+1}) - (\hat{r}_{Bt} - \hat{P}_{BHt} + \hat{P}^e_{BHt+1}) \\
&\quad +(\hat{P}^e_{BFt+1} - \hat{P}^e_{AHt+1}) - (\hat{P}_{BFt} - \hat{P}_{AHt}) \\
&= (\hat{r}_{At} - \hat{r}_{Bt}) + (\hat{P}_{BHt} - \hat{P}_{BFt}) - (\hat{P}^e_{BHt+1} - \hat{P}^e_{BFt+1}) \\
&= (\hat{r}_{At} - \hat{r}_{Bt}) + \hat{S}_{At} - \hat{S}^e_{At+1} \qquad (73)
\end{aligned}$$

Log-linearisation of home wealth allocation (20):

$$\begin{aligned}
B_{At} &= \alpha Z^{\phi}_{At} A_{At} + (1 - \alpha Z^{\phi}_{Bt}) A_{Bt} E_t \\
\frac{B_{At}}{P_{AHt}} &= \alpha Z^{\phi}_{At} \frac{A_{At}}{P_{AHt}} + (1 - \alpha Z^{\phi}_{Bt}) \frac{A_{Bt}}{P_{AHt}} \frac{P_{BHt}}{P_{BHt}} \frac{P_{AHt}}{P_{BFt}} \\
\tilde{B}_{At} &= \alpha Z^{\phi}_{At} \tilde{A}_{At} + (1 - \alpha Z^{\phi}_{Bt}) \tilde{A}_{Bt} S_{At} \\
\tilde{B}_A \hat{B}_{At} &= \alpha\phi \hat{Z}_{At} \tilde{A}_A + \alpha \tilde{A}_A \hat{A}_{At} + (1 - \alpha\phi \hat{Z}_{Bt}) \tilde{A}_B \\
&\quad + (1-\alpha) \tilde{A}_B \hat{A}_{Bt} + (1-\alpha) \tilde{A}_B \hat{S}_{At} \\
\hat{B}_{At} &= \alpha\phi \hat{Z}_{At} + \alpha \hat{A}_{At} + (1 - \alpha\phi \hat{Z}_{Bt}) + (1-\alpha) \hat{A}_{Bt} + (1-\alpha) \hat{S}_{At} \\
\hat{B}_{At} &= 2\alpha\phi \hat{Z}_{At} + (2\alpha - 1) \hat{A}_{At} + 1 + (1-\alpha) \hat{S}_{At} \\
\hat{B}_{At} &= 2\alpha\phi \hat{Z}_{At} + (2\alpha - 1)(\hat{B}_{At} - \hat{F}_{At}) + 1 + (1-\alpha) \hat{S}_{At} \\
2\alpha\phi \hat{Z}_{At} &= 2(1-\alpha) \hat{B}_{At} + (2\alpha - 1) \hat{F}_{At} - (1-\alpha) \hat{S}_{At} - 1 \qquad (74)
\end{aligned}$$

Loglinearisation of the home aggregate demand (26):

$$\begin{aligned}
Y_A \hat{Y}_{At} &= \mu C_A (\hat{C}_{At} + \hat{\mu}_{At} - \eta(\hat{P}_{AHt} - \hat{P}_{At})) + (1-\mu) C_B (\hat{C}_{Bt} - \eta(\hat{P}_{BFt} - \hat{P}_t)) \\
&\quad - \mu C_B \hat{\mu}_{Bt} + G_A \hat{G}_{At} \\
\hat{Y}_{At} &= \mu\theta(\hat{C}_{At} - \eta(\hat{P}_{AHt} - \hat{P}_{At})) + (1-\mu)\theta(\hat{C}_{Bt} - \eta(\hat{P}_{BFt} - \hat{P}_t)) \\
&\quad + \mu\theta(\hat{\mu}_{At} - \hat{\mu}_{Bt}) + (1-\theta) \hat{G}_{At} \\
\hat{Y}_{At} &= \mu\theta(\hat{C}_{At} + \eta(1-\mu) \hat{S}_{At}) + (1-\mu)\theta(\hat{C}_{Bt} + \eta\mu \hat{S}_{At}) + (1-\theta) \hat{G}_{At} + \mu\theta(\hat{\mu}_{At} - \hat{\mu}_{Bt}) \\
\hat{Y}_{At} &= \mu\theta \hat{C}_{At} + (1-\mu)\theta \hat{C}_{Bt} + 2\mu(1-\mu)\theta\eta \hat{S}_{At} + (1-\theta) \hat{G}_{At} + \mu\theta(\hat{\mu}_{At} - \hat{\mu}_{Bt}) \qquad (75)
\end{aligned}$$

Log-linearisation of the Euler equation (7):

$$\begin{aligned}
0 &= \hat{\rho}_{At} + \hat{C}_{At} + \hat{P}_{At} - \hat{C}_{At+1} - \hat{P}_{At+1} \\
0 &= 1 + \hat{i}_{At} - (1-\alpha) \hat{Z}_{At} + \hat{C}_{At} - \hat{C}_{At+1} \\
&\quad + (\hat{P}_{AHt} + (1-\mu) \hat{S}_{At}) - (\hat{P}_{AHt+1} + (1-\mu) \hat{S}_{At+1}) \\
0 &= 1 + \hat{r}_{At} - (1-\alpha) \hat{Z}_{At} + \hat{C}_{At} - \hat{C}_{At+1} + (1-\mu)(\hat{S}_{At} - \hat{S}_{At+1}) \\
\hat{C}_{At} &= \hat{C}_{At+1} - \hat{r}_{At} + (1-\alpha) \hat{Z}_{At} + (1-\mu)(\hat{S}_{At+1} - \hat{S}_{At}) - 1 \qquad (76)
\end{aligned}$$

Log-linearisation of the home fiscal constraint (18):

$$\begin{aligned}
B_{At+1} &= (1+i_{At})(B_{At} + P_{AHt}G_{At} - \tau P_{AHt}Y_{At}) \\
\frac{B_{At+1}}{P_{AHt+1}} &= (1+i_{At})\frac{P_{AHt}}{P_{AHt+1}}\left(\frac{B_{At} + P_{AHt}G_{At} - \tau P_{AHt}Y_{At}}{P_{AHt}}\right) \\
\tilde{B}_{At+1} &= (1+r_{At})(\tilde{B}_{At} + G_{At} - \tau Y_{At}) \\
\tilde{B_A}\hat{B}_{At+1} &= (1+r_A)(\tilde{B}_A + G_A - \tau Y_A)\hat{r}_{At} + (1+r_A)(\tilde{B}_A\hat{B}_{At} + G_A\hat{G}_{At} - \tau Y_A\hat{Y}_{At}) \\
\tilde{B_A}\hat{B}_{At+1} &= \tilde{B_A}\hat{r}_{At} + \frac{1}{\beta}(\tilde{B}_A\hat{B}_{At} + G_A\hat{G}_{At} - \tau Y_A\hat{Y}_{At}) \\
\hat{B}_{At+1} &= \hat{r}_{At} + \frac{1}{\beta}\left(\hat{B}_{At} + \frac{(1-\theta)}{B}\hat{G}_{At} - \frac{\tau}{B}\hat{Y}_{At}\right) \qquad (77)
\end{aligned}$$

Log-linearisation of the consumer budget constraint (29):

$$\begin{aligned}
A_{At+1} &= (1+\rho_{At})(A_{At} + (1-\tau)P_{AHt}Y_{At} - P_{At}C_{At}) \\
\frac{A_{At+1}}{P_{AHt+1}} &= \underbrace{(1+i_{At})\left(\alpha_{At} + \frac{(1-\alpha_{At})}{Z_{At}}\right)}_{(1+\rho_{At})}\frac{P_{AHt}}{P_{AHt+1}}\left(\frac{A_{At} + (1-\tau)P_{AHt}Y_{At} - P_{At}C_{At}}{P_{AHt}}\right) \\
\tilde{A}_{At+1} &= (1+r_{At})\left(\alpha_{At} + \frac{(1-\alpha_{At})}{Z_{At}}\right)\left(\tilde{A}_{At} + (1-\tau)Y_{At} - \frac{P_{At}}{P_{AHt}}C_{At}\right)
\end{aligned}$$

$$\begin{aligned}
\therefore \tilde{A}_A\hat{A}_{At+1} &= \tilde{A}_A\hat{r}_{At} + \tilde{A}_A(1-(1-\alpha)\hat{Z}_{At}) + \\
&\quad (1+r_A)(\tilde{A}_A\hat{A}_{At} + (1-\tau)Y_A\hat{Y}_{At} - C_A(\hat{C}_{At} + \hat{P}_{At} - \hat{P}_{AHt})) \\
B\hat{A}_{At+1} &= B\hat{r}_{At} + B(1-(1-\alpha)\hat{Z}_{At}) + \frac{1}{\beta}\left(B\hat{A}_{At} + (1-\tau)\hat{Y}_{At} - \theta(\hat{C}_{At} + (1-\mu)\hat{S}_{At})\right) \\
\hat{A}_{At+1} &= \hat{r}_{At} + 1 - (1-\alpha)\hat{Z}_{At} + \frac{1}{\beta}\left(\hat{A}_{At} + \frac{(1-\tau)}{B}\hat{Y}_{At} - \frac{\theta}{B}\hat{C}_{At} - \frac{(1-\mu)\theta}{B}\hat{S}_{At}\right) \\
\hat{F}_{At+1} &= (1-\alpha)\hat{Z}_{At} - 1 \\
&\quad + \frac{1}{\beta}\left(\hat{F}_{At} + \frac{(1-\theta)}{B}\hat{G}_{At} - \frac{1}{B}\hat{Y}_{At} + \frac{\theta}{B}\hat{C}_{At} + \frac{(1-\mu)\theta}{B}\hat{S}_{At}\right) \qquad (78)
\end{aligned}$$

where the last equality combines with the log-fiscal constraint (77). Then we can combine this with the log output equation (75) to get:

$$\hat{F}_{At+1} = (1-\alpha)\hat{Z}_{At} - 1 + \frac{1}{\beta}\left(\hat{F}_{At} + \frac{\theta}{B}(1-\mu)(\hat{C}_{At} - \hat{C}_{Bt}) - \frac{(1-\mu)\theta}{B}(2\mu\eta - 1)\hat{S}_{At}\right) \qquad (79)$$